Chapter 4
Trigonometric Functions

Section 4.1

Check Point Exercises

1. The radian measure of a central angle is the length of the intercepted arc, *s*, divided by the circle's radius, *r*. The length of the intercepted arc is 42 feet: $s = 42$ feet. The circle's radius is 12 feet: $r = 12$ feet. Now use the formula for radian measure to find the radian measure of θ.

$$\theta = \frac{s}{r} = \frac{42 \text{ feet}}{12 \text{ feet}} = 3.5$$

Thus, the radian measure of θ is 3.5

2. **a.** $60° = 60° \cdot \frac{\pi \text{ radians}}{180°} = \frac{60\pi}{180}$ radians

$= \frac{\pi}{3}$ radians

b. $270° = 270° \cdot \frac{\pi \text{ radians}}{180°} = \frac{270\pi}{180}$ radians

$= \frac{3\pi}{2}$ radians

c. $-300° = -300° \cdot \frac{\pi \text{ radians}}{180°} = \frac{-300\pi}{180}$ radians

$= -\frac{5\pi}{3}$ radians

3. **a.** $\frac{\pi}{4}$ radians $= \frac{\pi \text{ radians}}{4} \cdot \frac{180^{\text{o}}}{\pi \text{ radians}}$

$= \frac{180^{\text{o}}}{4} = 45^{\text{o}}$

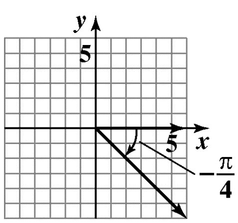
b. $-\frac{4\pi}{3}$ radians $= -\frac{4\pi \text{ radians}}{3} \cdot \frac{180^{\text{o}}}{\pi}$

$= -\frac{4 \cdot 180^{\text{o}}}{3} = -240^{\text{o}}$

c. 6 radians $= 6 \text{ radians} \cdot \frac{180^{\text{o}}}{\pi \text{ radians}}$

$= \frac{6 \cdot 180^{\text{o}}}{\pi} \approx 343.8^{\text{o}}$

4. **a.**



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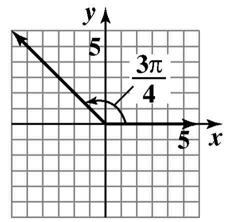

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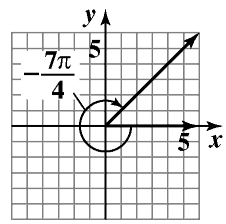

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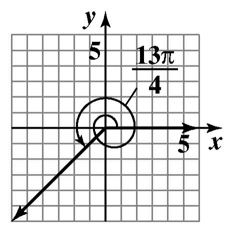

5. **a.** For a 400º angle, subtract 360º to find a positive coterminal angle.
$400^{\text{o}} - 360^{\text{o}} = 40^{\text{o}}$

b. For a –135º angle, add 360º to find a positive coterminal angle.
$-135^{\text{o}} + 360^{\text{o}} = 225^{\text{o}}$

6. **a.** $\frac{13\pi}{5} - 2\pi = \frac{13\pi}{5} - \frac{10\pi}{5} = \frac{3\pi}{5}$

b. $-\frac{\pi}{15} + 2\pi = -\frac{\pi}{15} + \frac{30\pi}{15} = \frac{29\pi}{15}$

7. **a.** $855° - 360° \cdot 2 = 855° - 720° = 135°$

b. $\frac{17\pi}{3} - 2\pi \cdot 2 = \frac{17\pi}{3} - 4\pi$

$= \frac{17\pi}{3} - \frac{12\pi}{3} = \frac{5\pi}{3}$

c. $-\frac{25\pi}{6} + 2\pi \cdot 3 = -\frac{25\pi}{6} + 6\pi$

$= -\frac{25\pi}{6} + \frac{36\pi}{6} = \frac{11\pi}{6}$

8. The formula $s = r\theta$ can only be used when θ is expressed in radians. Thus, we begin by converting 45° to radians. Multiply by $\frac{\pi \text{ radians}}{180°}$.

$$45° = 45° \cdot \frac{\pi \text{ radians}}{180°} = \frac{45}{180}\pi \text{ radians}$$
$$= \frac{\pi}{4} \text{ radians}$$

Now we can use the formula $s = r\theta$ to find the length of the arc. The circle's radius is 6 inches : $r = 6$ inches. The measure of the central angle in radians is $\frac{\pi}{4} : \theta = \frac{\pi}{4}$. The length of the arc intercepted by this central angle is

$$s = r\theta$$
$$= (6 \text{ inches})\left(\frac{\pi}{4}\right)$$
$$= \frac{6\pi}{4} \text{ inches}$$
$$= \frac{3\pi}{2} \text{ inches}$$
$$\approx 4.71 \text{ inches.}$$

9. We are given ω, the angular speed.
$\omega = 45$ revolutions per minute
We use the formula $v = r\omega$ to find v, the linear speed. Before applying the formula, we must express ω in radians per minute.

$$\omega = \frac{45 \text{ revolutions}}{1 \text{ minute}} \cdot \frac{2\pi \text{ radians}}{1 \text{ revolution}}$$
$$= \frac{90\pi \text{ radians}}{1 \text{ minute}}$$

The angular speed of the propeller is 90π radians per minute. The linear speed is

$$v = r\omega$$
$$= 1.5 \text{ inches} \cdot \frac{90\pi}{1 \text{ minute}}$$
$$= \frac{135\pi \text{ inches}}{\text{minute}}$$
$$= 135\pi \frac{\text{inches}}{\text{minute}}$$
$$\approx 424 \frac{\text{inches}}{\text{minute}}$$

The linear speed is 135π inches per minute, which is approximately 424 inches per minute.

Concept and Vocabulary Check 4.1

1. origin; x-axis

2. counterclockwise; clockwise

3. acute; right; obtuse; straight

4. $\frac{s}{r}$

5. $\frac{\pi}{180°}$

6. $\frac{180°}{\pi}$

7. coterminal; 360°; 2π

8. $r\theta$

9. false

10. $r\omega$; angular

Exercise Set 4.1

1. obtuse

2. obtuse

3. acute

4. acute

5. straight

6. right

7. $\theta = \frac{s}{r} = \frac{40 \text{ inches}}{10 \text{ inches}} = 4 \text{ radians}$

8. $\theta = \frac{s}{r} = \frac{30 \text{ feet}}{5 \text{ feet}} = 6 \text{ radians}$

9. $\theta = \frac{s}{r} = \frac{8 \text{ yards}}{6 \text{ yards}} = \frac{4}{3} \text{ radians}$

10. $\theta = \frac{s}{r} = \frac{18 \text{ yards}}{8 \text{ yards}} = 2.25 \text{ radians}$

11. $\theta = \frac{s}{r} = \frac{400 \text{ centimeters}}{100 \text{ centimeters}} = 4 \text{ radians}$

12. $\theta = \frac{s}{r} = \frac{600 \text{ centimeters}}{100 \text{ centimeters}} = 6 \text{ radians}$

13. $45° = 45° \cdot \frac{\pi \text{ radians}}{180°}$
$= \frac{45\pi}{180} \text{ radians}$
$= \frac{\pi}{4} \text{ radians}$

14. $18° = 18° \cdot \frac{\pi \text{ radians}}{180°}$
$= \frac{18\pi}{180} \text{ radians}$
$= \frac{\pi}{10} \text{ radians}$

15. $135° = 135° \cdot \frac{\pi \text{ radians}}{180°}$
$= \frac{135\pi}{180} \text{ radians}$
$= \frac{3\pi}{4} \text{ radians}$

16. $150° = 150° \cdot \frac{\pi \text{ radians}}{180°}$
$= \frac{150\pi}{180} \text{ radians}$
$= \frac{5\pi}{6} \text{ radians}$

17. $300° = 300° \cdot \frac{\pi \text{ radians}}{180°}$
$= \frac{300\pi}{180} \text{ radians}$
$= \frac{5\pi}{3} \text{ radians}$

18. $330° = 330° \cdot \frac{\pi \text{ radians}}{180°}$
$= \frac{330\pi}{180} \text{ radians}$
$= \frac{11\pi}{6} \text{ radians}$

19. $-225° = -225° \cdot \frac{\pi \text{ radians}}{180°}$
$= -\frac{225\pi}{180} \text{ radians}$
$= -\frac{5\pi}{4} \text{ radians}$

20. $-270° = -270° \cdot \frac{\pi \text{ radians}}{180°}$
$= -\frac{270\pi}{180} \text{ radians}$
$= -\frac{3\pi}{2} \text{ radians}$

21. $\frac{\pi}{2} \text{ radians} = \frac{\pi \text{ radians}}{2} \cdot \frac{180^{\text{o}}}{\pi \text{ radians}}$
$= \frac{180^{\text{o}}}{2}$
$= 90^{\text{o}}$

22. $\frac{\pi}{9} \text{ radians} = \frac{\pi \text{ radians}}{9} \cdot \frac{180^{\text{o}}}{\pi \text{ radians}}$
$= \frac{180^{\text{o}}}{9} = 20^{\text{o}}$

23. $\frac{2\pi}{3} \text{ radians} = \frac{2\pi \text{ radians}}{3} \cdot \frac{180^{\text{o}}}{\pi \text{ radians}}$
$= \frac{2 \cdot 180^{\text{o}}}{3}$
$= 120^{\text{o}}$

24. $\frac{3\pi \text{ radians}}{4} \cdot \frac{180^{0}}{\pi \text{ radians}} = \frac{3 \cdot 180^{\text{o}}}{4} = 135^{\text{o}}$

25. $\frac{7\pi}{6} \text{ radians} = \frac{7\pi \text{ radians}}{6} \cdot \frac{180^{\text{o}}}{\pi \text{ radians}}$
$= \frac{7 \cdot 180^{\text{o}}}{6}$
$= 210^{\text{o}}$

26. $\frac{11\pi \text{ radians}}{6} \cdot \frac{180^{\text{o}}}{\pi \text{ radians}} = \frac{11 \cdot 180^{\text{o}}}{6} = 330^{\text{o}}$

27. $-3\pi \text{ radians} = -3\pi \text{ radians} \cdot \dfrac{180^{\circ}}{\pi \text{ radians}}$
$$= -3 \cdot 180^{\circ}$$
$$= -540^{\circ}$$

28. $-4\pi \text{ radians } \cdot \dfrac{180^{\circ}}{\pi \text{ radians}} = -4 \cdot 180^{\circ} = -720^{\circ}$

29. $18° = 18° \cdot \dfrac{\pi \text{ radians}}{180°}$
$$= \frac{18\pi}{180} \text{ radians}$$
$$\approx 0.31 \text{ radians}$$

30. $76° = 76° \cdot \dfrac{\pi \text{ radians}}{180°}$
$$= \frac{76\pi}{180} \text{ radians}$$
$$\approx 1.33 \text{ radians}$$

31. $-40° = -40° \cdot \dfrac{\pi \text{ radians}}{180°}$
$$= -\frac{40\pi}{180} \text{ radians}$$
$$\approx -0.70 \text{ radians}$$

32. $-50° = -50° \cdot \dfrac{\pi \text{ radians}}{180°}$
$$= -\frac{50\pi}{180} \text{ radians}$$
$$\approx -0.87 \text{ radians}$$

33. $200° = 200° \cdot \dfrac{\pi \text{ radians}}{180°}$
$$= \frac{200\pi}{180} \text{ radians}$$
$$\approx 3.49 \text{ radians}$$

34. $250° = 250° \cdot \dfrac{\pi \text{ radians}}{180°}$
$$= \frac{250\pi}{180} \text{ radians}$$
$$\approx 4.36 \text{ radians}$$

35. $2 \text{ radians} = 2 \text{ radians} \cdot \dfrac{180^{\circ}}{\pi \text{ radians}}$
$$= \frac{2 \cdot 180^{\circ}}{\pi}$$
$$\approx 114.59^{\circ}$$

36. $3 \text{ radians} \cdot \dfrac{180^{\circ}}{\pi \text{ radians}} = \dfrac{3 \cdot 180^{\circ}}{\pi} \approx 171.89^{\circ}$

37. $\dfrac{\pi}{13} \text{ radians} = \dfrac{\pi \text{ radians}}{13} \cdot \dfrac{180^{\circ}}{\pi \text{ radians}}$
$$= \frac{180^{\circ}}{13}$$
$$\approx 13.85^{\circ}$$

38. $\dfrac{\pi}{17} \text{radians} \cdot \dfrac{180^{\circ}}{\pi \text{radians}} = \dfrac{180^{\circ}}{17} \approx 10.59^{\circ}$

39. $-4.8 \text{ radians} = -4.8 \text{ radians} \cdot \dfrac{180^{\circ}}{\pi \text{ radians}}$
$$= \frac{-4.8 \cdot 180^{\circ}}{\pi}$$
$$\approx -275.02^{\circ}$$

40. $-5.2 \text{ radians} \cdot \dfrac{180^{\circ}}{\pi \text{radians}} = \dfrac{-5.2 \cdot 180^{\circ}}{\pi}$
$$\approx -297.94^{\circ}$$

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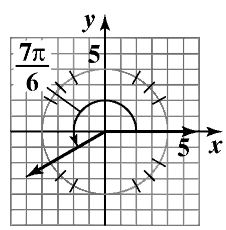

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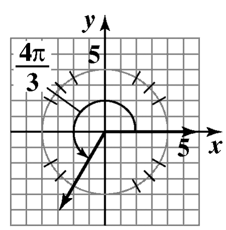

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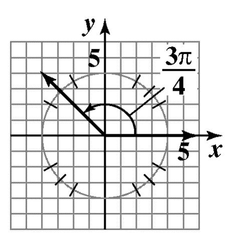

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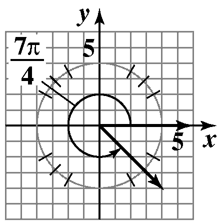

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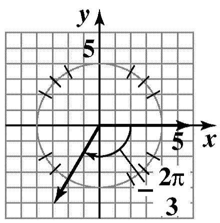

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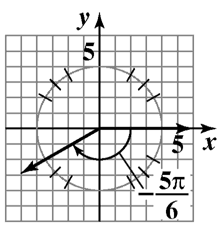

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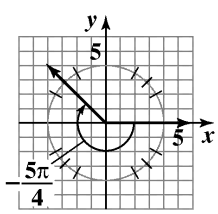

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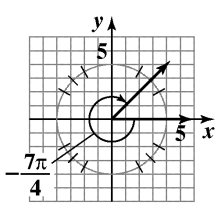

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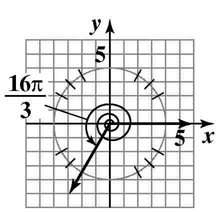

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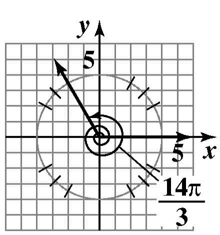

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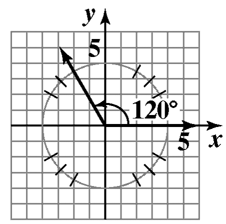

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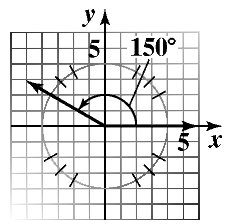

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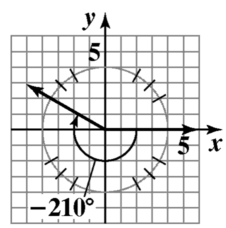

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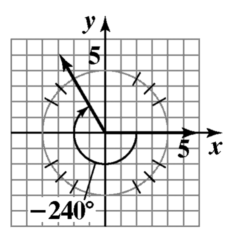

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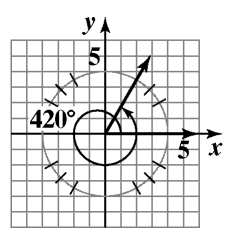

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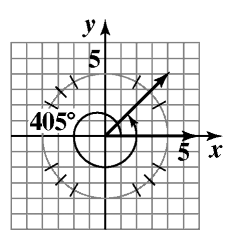


57. $395° - 360° = 35°$

58. $415° - 360° = 55°$

59. $-150° + 360° = 210°$

60. $-160° + 360° = 200°$

61. $-765°+360°\cdot 3=-765°+1080°=315°$

62. $-760°+360°\cdot 3=-760°+1080°=320°$

63. $\frac{19\pi}{6}-2\pi=\frac{19\pi}{6}-\frac{12\pi}{6}=\frac{7\pi}{6}$

64. $\frac{17\pi}{5}-2\pi=\frac{17\pi}{5}-\frac{10\pi}{5}=\frac{7\pi}{5}$

65. $\frac{23\pi}{5}-2\pi\cdot 2=\frac{23\pi}{5}-4\pi=\frac{23\pi}{5}-\frac{20\pi}{5}=\frac{3\pi}{5}$

66. $\frac{25\pi}{6}-2\pi\cdot 2=\frac{25\pi}{6}-4\pi=\frac{25\pi}{6}-\frac{24\pi}{6}=\frac{\pi}{6}$

67. $-\frac{\pi}{50}+2\pi=-\frac{\pi}{50}+\frac{100\pi}{50}=\frac{99\pi}{50}$

68. $-\frac{\pi}{40}+2\pi=-\frac{\pi}{40}+\frac{80\pi}{40}=\frac{79\pi}{40}$

69. $-\frac{31\pi}{7}+2\pi\cdot 3=-\frac{31\pi}{7}+6\pi$

$=-\frac{31\pi}{7}+\frac{42\pi}{7}=\frac{11\pi}{7}$

70. $-\frac{38\pi}{9}+2\pi\cdot 3=-\frac{38\pi}{9}+6\pi$

$=-\frac{38\pi}{9}+\frac{54\pi}{9}=\frac{16\pi}{9}$

71. $r=12$ inches, $\theta=45°$

Begin by converting 45° to radians, in order to use the formula $s=r\theta$.

$45°=45°\cdot\frac{\pi\text{ radians}}{180°}=\frac{\pi}{4}$ radians

Now use the formula $s=r\theta$.

$s=r\theta=12\cdot\frac{\pi}{4}=3\pi$ inches ≈ 9.42 inches

72. $r=16$ inches, $\theta=60°$

Begin by converting 60° to radians, in order to use the formula $s=r\theta$.

$60°=60°\cdot\frac{\pi\text{ radians}}{180°}=\frac{\pi}{3}$ radians

Now use the formula $s=r\theta$.

$s=r\theta=16\cdot\frac{\pi}{3}=\frac{16\pi}{3}$ inches ≈ 16.76 inches

73. $r=8$ feet, $\theta=225°$

Begin by converting 225° to radians, in order to use the formula $s=r\theta$.

$225°=225°\cdot\frac{\pi\text{ radians}}{180°}=\frac{5\pi}{4}$ radians

Now use the formula $s=r\theta$.

$s=r\theta=8\cdot\frac{5\pi}{4}=10\pi$ feet ≈ 31.42 feet

74. $r=9$ yards, $\theta=315°$

Begin by converting 315° to radians, in order to use the formula $s=r\theta$.

$315°=315°\cdot\frac{\pi\text{ radians}}{180°}=\frac{7\pi}{4}$ radians

Now use the formula $s=r\theta$.

$s=r\theta=9\cdot\frac{7\pi}{4}=\frac{63\pi}{4}$ yards ≈ 49.48 yards

75. 6 revolutions per second

$=\frac{6\text{ revolutions}}{1\text{ second}}\cdot\frac{2\pi\text{ radians}}{1\text{ revolutions}}=\frac{12\pi\text{ radians}}{1\text{ seconds}}$

$=12\pi$ radians per second

76. 20 revolutions per second

$=\frac{20\text{ revolutions}}{1\text{ second}}\cdot\frac{2\pi\text{ radians}}{1\text{ revolution}}=\frac{40\pi\text{ radians}}{1\text{ second}}$

$=40\pi$ radians per second

77. $-\frac{4\pi}{3}$ and $\frac{2\pi}{3}$

78. $-\frac{7\pi}{6}$ and $\frac{5\pi}{6}$

79. $-\frac{3\pi}{4}$ and $\frac{5\pi}{4}$

80. $-\frac{\pi}{4}$ and $\frac{7\pi}{4}$

81. $-\frac{\pi}{2}$ and $\frac{3\pi}{2}$

82. $-\pi$ and π

83. $\frac{55}{60}\cdot 2\pi=\frac{11\pi}{6}$

84. $\frac{35}{60}\cdot 2\pi=\frac{7\pi}{6}$

85. 3 minutes and 40 seconds equals 220 seconds.

$$\frac{220}{60}\cdot 2\pi = \frac{22\pi}{3}$$

86. 4 minutes and 25 seconds equals 265 seconds.

$$\frac{265}{60}\cdot 2\pi = \frac{53\pi}{6}$$

87. First, convert to degrees.

$$\frac{1}{6}\text{ revolution} = \frac{1}{6}\text{ revolution}\cdot\frac{360°}{1\text{ revolution}}$$
$$= \frac{1}{6}\cdot 360° = 60°$$

Now, convert 60° to radians.

$$60° = 60°\cdot\frac{\pi\text{ radians}}{180°} = \frac{60\pi}{180}\text{ radians}$$
$$= \frac{\pi}{3}\text{ radians}$$

Therefore, $\frac{1}{6}$ revolution is equivalent to 60° or $\frac{\pi}{3}$ radians.

88. First, convert to degrees.

$$\frac{1}{3}\text{revolutions} = \frac{1}{3}\text{ revolutions}\cdot\frac{360^{\text{O}}}{1\text{ revolution}}$$
$$= \frac{1}{3}\cdot 360^{\text{O}} = 120^{\text{O}}$$

Now, convert 120° to radians.

$$120° = 120°\cdot\frac{\pi\text{ radians}}{180°} = \frac{120\pi}{180}\text{ radians} = \frac{2\pi}{3}\text{ radians}$$

Therefore, $\frac{1}{3}$ revolution is equivalent to 120° or $\frac{2\pi}{3}$ radians.

89. The distance that the tip of the minute hand moves is given by its arc length, s. Since $s = r\theta$, we begin by finding r and θ. We are given that $r = 8$ inches. The minute hand moves from 12 to 2 o'clock, or $\frac{1}{6}$ of a complete revolution. The formula $s = r\theta$ can only be used when θ is expressed in radians. We must convert $\frac{1}{6}$ revolution to radians.

$$\frac{1}{6}\text{ revolution} = \frac{1}{6}\text{ revolution}\cdot\frac{2\pi\text{ radians}}{1\text{ revolution}}$$
$$= \frac{\pi}{3}\text{ radians}$$

The distance the tip of the minute hand moves is

$$s = r\theta = (8\text{ inches})\left(\frac{\pi}{3}\right) = \frac{8\pi}{3}\text{ inches} \approx 8.38\text{ inches.}$$

90. The distance that the tip of the minute hand moves is given by its arc length, s. Since $s = r\theta$, we begin by finding r and θ. We are given that $r = 6$ inches. The minute hand moves from 12 to 4 o'clock, or $\frac{1}{3}$ of a complete revolution. The formula $s = r\theta$ can only be used when θ is expressed in radians. We must convert $\frac{1}{3}$ revolution to radians.

$$\frac{1}{3}\text{ revolution} = \frac{1}{3}\text{revolution}\cdot\frac{2\pi\text{ radians}}{1\text{ revolution}}$$
$$= \frac{2\pi}{3}\text{ radians}$$

The distance the tip of the minute hand moves is

$$s = r\theta = (6\text{ inches})\left(\frac{2\pi}{3}\right) = \frac{12\pi}{3}\text{ inches}$$
$$= 4\pi\text{ inches} \approx 12.57\text{ inches.}$$

91. The length of each arc is given by $s = r\theta$. We are given that $r = 24$ inches and $\theta = 90°$. The formula $s = r\theta$ can only be used when θ is expressed in radians.

$$90° = 90°\cdot\frac{\pi\text{ radians}}{180°} = \frac{90\pi}{180}\text{ radians}$$
$$= \frac{\pi}{2}\text{ radians}$$

The length of each arc is

$$s = r\theta = (24\text{ inches})\left(\frac{\pi}{2}\right) = 12\pi\text{ inches}$$
$$\approx 37.70\text{ inches.}$$

92. The distance that the wheel moves is given by $s = r\theta$. We are given that $r = 80$ centimeters and $\theta = 60°$. The formula $s = r\theta$ can only be used when θ is expressed in radians.

$$60° = 60°\cdot\frac{\pi\text{ radians}}{180°} = \frac{60\pi}{180}\text{ radians}$$
$$= \frac{\pi}{3}\text{ radians}$$

The length that the wheel moves is

$$s = r\theta = (80\text{ centimeters})\left(\frac{\pi}{3}\right) = \frac{80\pi}{3}\text{ centimeters}$$
$$\approx 83.78\text{ centimeters.}$$

93. Recall that $\theta = \frac{s}{r}$. We are given that s = 8000 miles and r = 4000 miles.

$$\theta = \frac{s}{r} = \frac{8000 \text{ miles}}{4000 \text{ miles}} = 2 \text{ radians}$$

Now, convert 2 radians to degrees.

$$2 \text{ radians} = 2 \text{ radians} \cdot \frac{180^{\circ}}{\pi \text{ radians}} \approx 114.59^{\circ}$$

94. Recall that $\theta = \frac{s}{r}$. We are given that s = 10,000 miles and r = 4000 miles.

$$\theta = \frac{s}{r} = \frac{10{,}000 \text{ miles}}{4000 \text{ miles}} = 2.5 \text{ radians}$$

Now, convert 2.5 radians to degrees.

$$2.5 \text{ radians} \cdot \frac{180^{\circ}}{2\pi \text{ radians}} \approx 143.24^{\circ}$$

95. Recall that $s = r\theta$. We are given that r = 4000 miles and $\theta = 30°$. The formula $s = r\theta$ can only be used when θ is expressed in radians.

$$30° = 30° \cdot \frac{\pi \text{ radians}}{180°} = \frac{30\pi}{180} \text{ radians}$$

$$= \frac{\pi}{6} \text{ radians}$$

$$s = r\theta = (4000 \text{ miles})\left(\frac{\pi}{6}\right) \approx 2094 \text{ miles}$$

To the nearest mile, the distance from A to B is 2094 miles.

96. Recall that $s = r\theta$. We are given that r = 4000 miles and $\theta = 10°$. We can only use the formula $s = r\theta$ when θ is expressed in radians.

$$10° = 10° \cdot \frac{\pi \text{ radians}}{180°} = \frac{10\pi}{180} \text{ radians}$$

$$= \frac{\pi}{18} \text{ radians}$$

$$s = r\theta = (4000 \text{ miles})\left(\frac{\pi}{18}\right) \approx 698 \text{ miles}$$

To the nearest mile, the distance from A to B is 698 miles.

97. Linear speed is given by $v = r\omega$. We are given that $\omega = \frac{\pi}{12}$ radians per hour and r = 4000 miles. Therefore,

$$v = r\omega = (4000 \text{ miles})\left(\frac{\pi}{12}\right)$$

$$= \frac{4000\pi}{12} \text{ miles per hour}$$

$$\approx 1047 \text{ miles per hour}$$

The linear speed is about 1047 miles per hour.

98. Linear speed is given by $v = r\omega$. We are given that r = 25 feet and the wheel rotates at 2 revolutions per minute. We need to convert 2 revolutions per minute to radians per minute.

2 revolutions per minute

$$= 2 \text{ revolutions per minute} \cdot \frac{2\pi \text{ radians}}{1 \text{ revolution}}$$

$$= 4\pi \text{ radians per minute}$$

$$v = r\omega = (25 \text{ feet})(4\pi) \approx 314 \text{ feet per minute}$$

The linear speed of the Ferris wheel is about 314 feet per minute.

99. Linear speed is given by $v = r\omega$. We are given that r = 12 feet and the wheel rotates at 20 revolutions per minute.

20 revolutions per minute

$$= 20 \text{ revolutions per minute} \cdot \frac{2\pi \text{ radians}}{1 \text{ revolution}}$$

$$= 40\pi \text{ radians per minute}$$

$$v = r\omega = (12 \text{ feet})(40\pi)$$

$$\approx 1508 \text{ feet per minute}$$

The linear speed of the wheel is about 1508 feet per minute.

100. Begin by converting 2.5 revolutions per minute to radians per minute.

2.5 revolutions per minute

$$= 2.5 \text{ revolutions per minute} \cdot \frac{2\pi \text{ radians}}{1 \text{ revolution}}$$

$$= 5\pi \text{ radians per minute}$$

The linear speed of the animals in the outer rows is

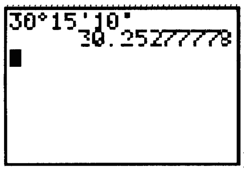
$$v = r\omega = (20 \text{ feet})(5\pi) \approx 100 \text{ feet per minute}$$

The linear speed of the animals in the inner rows is

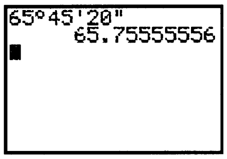
$$v = r\omega = (10 \text{ feet})(5\pi) \approx 50 \text{ feet per minute}$$

The difference is $100\pi - 50\pi = 50\pi$ feet per minute or about 157 feet per minute.

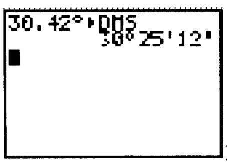
101. – 112. Answers may vary.

113. 

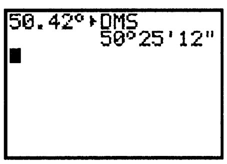
$30.25°$

114. 

$65.76°$

115. 

30° 25'12"

116. 

50° 25' 12"

117. does not make sense; Explanations will vary. Sample explanation: Angles greater than π will exceed a straight angle.

118. does not make sense; Explanations will vary. Sample explanation: It is possible for π to be used in an angle measured using degrees.

119. makes sense

120. does not make sense; Explanations will vary. Sample explanation: That will not be possible if the angle is a multiple of 2π.

121. A right angle measures 90° and

$$90° = \frac{\pi}{2} \text{ radians} \approx 1.57 \text{ radians.}$$

If $\theta = \frac{3}{2}$ radians $= 1.5$ radians, θ is smaller than a right angle.

122. $s = r\theta$

Begin by changing $\theta = 20°$ to radians.

$$20° = 20° \cdot \frac{\pi}{180°} = \frac{\pi}{9} \text{ radians}$$

$$100 = \frac{\pi}{9} r$$

$$r = \frac{900}{\pi} \approx 286 \text{ miles}$$

To the nearest mile, a radius of 286 miles should be used.

123. $s = r\theta$

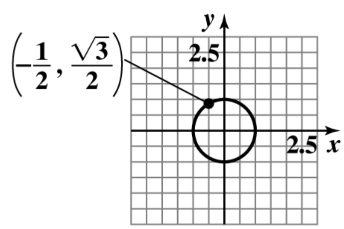
Begin by changing $\theta = 26°$ to radians.

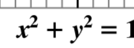
$$26° = 26° \cdot \frac{\pi}{180°} = \frac{13\pi}{90} \text{ radians}$$

$$s = 4000 \cdot \frac{13\pi}{90}$$

$$\approx 1815 \text{ miles}$$

To the nearest mile, Miami, Florida is 1815 miles north of the equator.

124. 



125. domain: $\{x \mid -1 \le x \le 1\}$ or $[-1,1]$

range: $\{y \mid -1 \le y \le 1\}$ or $[-1,1]$

126. $x = -\frac{1}{2}; \quad y = \frac{\sqrt{3}}{2}$

$$\frac{x}{y} = \frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}} = -\frac{1}{\sqrt{3}} = -\frac{1}{\sqrt{3}}\frac{\sqrt{3}}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$$

Section 4.2

Check Point Exercises

1. $P\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$\sin t = y = \frac{1}{2}$

$\cos t = x = \frac{\sqrt{3}}{2}$

$\tan t = \frac{y}{x} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{\sqrt{3}}{3}$

$\csc t = \frac{1}{y} = 2$

$\sec t = \frac{1}{x} = \frac{2\sqrt{3}}{3}$

$\cot t = \frac{x}{y} = \sqrt{3}$

2. The point P on the unit circle that corresponds to $t = \pi$ has coordinates (–1, 0). Use $x = -1$ and $y = 0$ to find the values of the trigonometric functions.

$\sin \pi = y = 0$

$\cos \pi = x = -1$

$\tan \pi = \frac{y}{x} = \frac{0}{-1} = 0$

$\sec \pi = \frac{1}{x} = \frac{1}{-1} = -1$

$\cot \pi = \frac{x}{y} = \frac{-1}{0} = \text{undefined}$

$\csc \pi = \frac{1}{y} = \frac{1}{0} = \text{undefined}$

3. $t = \frac{\pi}{4}, P\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

$\csc \frac{\pi}{4} = \frac{1}{y} = \sqrt{2}$

$\sec \frac{\pi}{4} = \frac{1}{x} = \sqrt{2}$

$\cot \frac{\pi}{4} = \frac{x}{y} = \frac{\frac{1}{y}}{\frac{1}{\sqrt{2}}} = 1$

4. **a.** $\sec\left(-\frac{\pi}{4}\right) = \sec\left(\frac{\pi}{4}\right) = \sqrt{2}$

b. $\sin\left(-\frac{\pi}{4}\right) = -\sin\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

5. $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{2}{3}}{\frac{\sqrt{5}}{3}}$

$= \frac{2}{3} \cdot \frac{3}{\sqrt{5}} = \frac{2}{\sqrt{5}}$

$= \frac{2}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$

$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{2}{3}} = \frac{3}{2}$

$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{\sqrt{5}}{3}} = \frac{3}{\sqrt{5}}$

$= \frac{3}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{3\sqrt{5}}{5}$

$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{\frac{2}{\sqrt{5}}} = \frac{\sqrt{5}}{2}$

6. $\sin t = \frac{1}{2}, 0 \le t < \frac{\pi}{2}$

$\sin^2 t + \cos^2 t = 1$

$\left(\frac{1}{2}\right)^2 + \cos^2 t = 1$

$\cos^2 t = 1 - \frac{1}{4}$

$\cos t = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$

Because $0 \le t < \frac{\pi}{2}, \cos t$ is positive.

7. a. $\cot\frac{5\pi}{4} = \cot\left(\frac{\pi}{4}+\pi\right) = \cot\frac{\pi}{4} = 1$

b. $\cos\left(-\frac{9\pi}{4}\right) = \cos\left(-\frac{9\pi}{4}+4\pi\right)$
$= \cos\frac{7\pi}{4}$
$= \frac{\sqrt{2}}{2}$

8. a. $\sin\frac{\pi}{4} \approx 0.7071$

b. $\csc 1.5 \approx 1.0025$

Concept and Vocabulary Check 4.2

1. intercepted are

2. cosine; sine

3. sine; cosine; $(-\infty,\infty)$

4. 1; -1; $[-1,1]$

5. $\frac{\sqrt{2}}{2}$; $\frac{\sqrt{2}}{2}$; 1

6. cos t; sec t; even

7. $-\sin t$; $-\csc t$; $-\tan t$; $-\cot t$; odd

8. $\sin t$; $\cos t$; $\tan t$

9. $\tan t$; $\cot t$

10. 1; $\sec^2 t$; $\csc^2 t$

11. periodic; period

12. $\sin t$; $\cos t$; periodic; 2π

13. $\tan t$; $\cot t$; periodic; π

Exercise Set 4.2

1. The point P on the unit circle has coordinates $\left(-\frac{15}{17},\frac{8}{17}\right)$. Use $x=-\frac{15}{17}$ and $y=\frac{8}{17}$ to find the values of the trigonometric functions.

$\sin t = y = \frac{8}{17}$

$\cos t = x = -\frac{15}{17}$

$\tan t = \frac{y}{x} = \frac{\frac{8}{17}}{-\frac{15}{17}} = -\frac{8}{15}$

$\csc t = \frac{1}{y} = \frac{17}{8}$

$\sec t = \frac{1}{x} = -\frac{17}{15}$

$\cot t = \frac{x}{y} = -\frac{15}{8}$

2. The point P on the unit circle has coordinates $\left(-\frac{5}{13},-\frac{12}{13}\right)$ Use $x=-\frac{5}{13}$ and $y=-\frac{12}{13}$ to find the values of the trigonometric functions.

$\sin t = y = -\frac{12}{13}$

$\cos t = x = -\frac{5}{13}$

$\tan t = \frac{y}{x} = \frac{-\frac{12}{13}}{-\frac{5}{13}} = \frac{12}{5}$

$\csc t = \frac{1}{y} = -\frac{13}{12}$

$\sec t = \frac{1}{x} = -\frac{13}{5}$

$\cot t = \frac{x}{y} = \frac{5}{12}$

3. The point P on the unit circle that corresponds to $t=-\frac{\pi}{4}$ has coordinates $\left(\frac{\sqrt{2}}{2},-\frac{\sqrt{2}}{2}\right)$. Use $x=\frac{\sqrt{2}}{2}$ and $y=-\frac{\sqrt{2}}{2}$ to find the values of the trigonometric functions.

$\sin t = y = -\frac{\sqrt{2}}{2}$

$\cos t = x = \frac{\sqrt{2}}{2}$

$\tan t = \frac{y}{x} = \frac{-\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = -1$

$\csc t = \frac{1}{y} = -\sqrt{2}$

$\sec t = \frac{1}{x} = \sqrt{2}$

$\cot t = \frac{x}{y} = -1$

4. The point P on the unit circle that corresponds to $t=\frac{3\pi}{4}$ has coordinates $\left(-\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right)$. Use $x=-\frac{\sqrt{2}}{2}$ and $y=\frac{\sqrt{2}}{2}$ to find the values of the trigonometric functions.

$\sin t = y = \frac{\sqrt{2}}{2}$

$\cos t = x = -\frac{\sqrt{2}}{2}$

$\tan t = \frac{y}{x} = \frac{\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}} = -1$

$\csc t = \frac{1}{y} = \sqrt{2}$

$\sec t = \frac{1}{x} = -\sqrt{2}$

$\cot t = \frac{x}{y} = -1$

5. $\sin\frac{\pi}{6} = \frac{1}{2}$

6. $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

7. $\cos\frac{5\pi}{6} = -\frac{\sqrt{3}}{2}$

8. $\cos\frac{2\pi}{3} = -\frac{1}{2}$

9. $\tan\pi = \frac{0}{-1} = 0$

10. $\tan 0 = \frac{0}{1} = 0$

11. $\csc\frac{7\pi}{6} = \frac{1}{-\frac{1}{2}} = -2$

12. $\csc\frac{4\pi}{3} = \frac{1}{-\frac{\sqrt{3}}{2}} = \frac{-2\sqrt{3}}{3}$

13. $\sec\frac{11\pi}{6} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2\sqrt{3}}{3}$

14. $\sec\frac{5\pi}{3} = \frac{1}{\frac{1}{2}} = 2$

15. $\sin\frac{3\pi}{2} = -1$

16. $\cos\frac{3\pi}{2} = 0$

17. $\sec\frac{3\pi}{2} = \text{undefined}$

18. $\tan\frac{3\pi}{2} = \text{undefined}$

19. **a.** $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

b. $\cos\left(-\frac{\pi}{6}\right) = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

20. **a.** $\cos\frac{\pi}{3} = \frac{1}{2}$

b. $\cos\left(-\frac{\pi}{3}\right) = \cos\frac{\pi}{3} = \frac{1}{2}$

21. a. $\sin\frac{5\pi}{6}=\frac{1}{2}$

b. $\sin\left(-\frac{5\pi}{6}\right)=-\sin\frac{5\pi}{6}=-\frac{1}{2}$

22. a. $\sin\frac{2\pi}{3}=\frac{\sqrt{3}}{2}$

b. $\sin\left(-\frac{2\pi}{3}\right)=-\sin\frac{2\pi}{3}=-\frac{\sqrt{3}}{2}$

23. a. $\tan\frac{5\pi}{3}=\frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}}=-\sqrt{3}$

b. $\tan\left(-\frac{5\pi}{3}\right)=-\tan\frac{5\pi}{3}=\sqrt{3}$

24. a. $\tan\frac{11\pi}{6}=\frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}}=-\frac{\sqrt{3}}{3}$

b. $\tan\left(-\frac{11\pi}{6}\right)=-\tan\frac{11\pi}{6}=\frac{\sqrt{3}}{3}$

25. $\sin t=\frac{8}{17}, \cos t=\frac{15}{17}$

$\tan t=\frac{\frac{8}{17}}{\frac{15}{17}}=\frac{8}{15}$

$\csc t=\frac{17}{8}$

$\sec t=\frac{17}{15}$

$\cot t=\frac{15}{8}$

26. $\sin t=\frac{3}{5}, \cos t=\frac{4}{5}$

$\tan t=\frac{\frac{3}{5}}{\frac{4}{5}}=\frac{3}{4}$

$\csc t=\frac{5}{3}$

$\sec t=\frac{5}{4}$

$\cot t=\frac{4}{3}$

27. $\sin t=\frac{1}{3}, \cos t=\frac{2\sqrt{2}}{3}$

$\tan t=\frac{\frac{1}{3}}{\frac{2\sqrt{2}}{3}}=\frac{\sqrt{2}}{4}$

$\csc t=3$

$\sec t=\frac{3\sqrt{2}}{4}$

$\cot t=2\sqrt{2}$

28. $\sin t=\frac{2}{3}, \cos t=\frac{\sqrt{5}}{3}$

$\tan t=\frac{\frac{2}{3}}{\frac{\sqrt{5}}{3}}=\frac{2\sqrt{5}}{5}$

$\csc t=\frac{3}{2}$

$\sec t=\frac{3\sqrt{5}}{5}$

$\cot t=\frac{\sqrt{5}}{2}$

29. $\sin t=\frac{6}{7}, 0\le t<\frac{\pi}{2}$

$\sin^2 t+\cos^2 t=1$

$\left(\frac{6}{7}\right)^2+\cos^2 t=1$

$\cos^2 t=1-\frac{36}{49}$

$\cos t=\sqrt{\frac{13}{49}}=\frac{\sqrt{13}}{7}$

Because $0\le t<\frac{\pi}{2}, \cos t$ is positive.

30. $\sin t=\frac{7}{8}, 0\le t<\frac{\pi}{2}$

$\sin^2 t+\cos^2 t=1$

$\left(\frac{7}{8}\right)^2+\cos^2 t=1$

$\cos^2 t=1-\frac{49}{64}$

$\cos t=\sqrt{\frac{15}{64}}=\frac{\sqrt{15}}{8}$

Because $0\le t<\frac{\pi}{2}, \cos t$ is positive.

31. $\sin t = \frac{\sqrt{39}}{8}, 0 \le t < \frac{\pi}{2}$

$$\sin^2 t + \cos^2 t = 1$$
$$\left(\frac{\sqrt{39}}{8}\right)^2 + \cos^2 t = 1$$
$$\cos^2 t = 1 - \frac{39}{64}$$
$$\cos t = \sqrt{\frac{25}{64}} = \frac{5}{8}$$

Because $0 \le t < \frac{\pi}{2}, \cos t$ is positive.

32. $\sin t = \frac{\sqrt{21}}{5}, 0 \le t < \frac{\pi}{2}$

$$\sin^2 t + \cos^2 t = 1$$
$$\left(\frac{\sqrt{21}}{5}\right)^2 + \cos^2 t = 1$$
$$\cos^2 t = 1 - \frac{21}{25}$$
$$\cos t = \sqrt{\frac{4}{25}} = \frac{2}{5}$$

Because $0 \le t < \frac{\pi}{2}, \cos t$ is positive.

33. $\sin 1.7 \csc 1.7 = \sin 1.7\left(\frac{1}{\sin 1.7}\right) = 1$

34. $\cos 2.3 \sec 2.3 = \cos 2.3\left(\frac{1}{\cos 2.3}\right) = 1$

35. $\sin^2 \frac{\pi}{6} + \cos^2 \frac{\pi}{2} = 1$ by the Pythagorean identity.

36. $\sin^2 \frac{\pi}{3} + \cos^2 \frac{\pi}{3} = 1$ because $\sin^2 t + \cos^2 t = 1$.

37. $\sec^2 \frac{\pi}{3} - \tan^2 \frac{\pi}{3} = 1$ because $1 + \tan^2 t = \sec^2 t$.

38. $\csc^2 \frac{\pi}{6} - \cot^2 \frac{\pi}{6} = 1$ because $1 + \cot^2 t = \csc^2 t$.

39. $\cos \frac{9\pi}{4} = \cos\left(\frac{\pi}{4} + 2\pi\right) = \cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$

40. $\csc \frac{9\pi}{4} = \csc\left(\frac{\pi}{4} + 2\pi\right) = \csc \frac{\pi}{4} = \sqrt{2}$

41. $\sin\left(-\frac{9\pi}{4}\right) = \sin\left(-\frac{9\pi}{4} + 4\pi\right) = \sin \frac{7\pi}{4} = -\frac{\sqrt{2}}{2}$

42. $\sec\left(-\frac{9\pi}{4}\right) = \sec\left(-\frac{9\pi}{4} + 4\pi\right) = \sec \frac{7\pi}{4} = \sqrt{2}$

43. $\tan \frac{5\pi}{4} = \tan\left(\frac{\pi}{4} + \pi\right) = \tan \frac{\pi}{4} = 1$

44. $\cot \frac{5\pi}{4} = \cot\left(\frac{\pi}{4} + \pi\right) = \cot \frac{\pi}{4} = 1$

45. $\cot\left(-\frac{5\pi}{4}\right) = \cot\left(\frac{3\pi}{4} - 2\pi\right) = \cot \frac{3\pi}{4} = -1$

46. $\tan\left(-\frac{9\pi}{4}\right) = \tan\left(-\frac{9\pi}{4} + 3\pi\right) = \tan \frac{3\pi}{4} = -1$

47. $-\tan\left(\frac{\pi}{4} + 15\pi\right) = -\tan \frac{\pi}{4} = -1$

48. $-\cot\left(\frac{\pi}{4} + 17\pi\right) = -\cot \frac{\pi}{4} = -1$

49. $\sin\left(-\frac{\pi}{4} - 1000\pi\right) = \sin\left(-\frac{\pi}{4} + 2\pi\right)$
$$= \sin \frac{7\pi}{4}$$
$$= -\frac{\sqrt{2}}{2}$$

50. $\sin\left(-\frac{\pi}{4} - 2000\pi\right) = \sin\left(-\frac{\pi}{4} + 2\pi\right)$
$$= \sin \frac{7\pi}{4}$$
$$= -\frac{\sqrt{2}}{2}$$

51. $\cos\left(-\frac{\pi}{4}-1000\pi\right)=\cos\left(-\frac{\pi}{4}+2\pi\right)$
$=\cos\frac{7\pi}{4}$
$=\frac{\sqrt{2}}{2}$

52. $\cos\left(-\frac{\pi}{4}-2000\pi\right)=\cos\left(-\frac{\pi}{4}+2\pi\right)$
$=\cos\frac{7\pi}{4}$
$=\frac{\sqrt{2}}{2}$

53. a. $\sin\frac{3\pi}{4}=\frac{\sqrt{2}}{2}$

b. $\sin\frac{11\pi}{4}=\sin\left(\frac{3\pi}{4}+2\pi\right)=\sin\frac{3\pi}{4}=\frac{\sqrt{2}}{2}$

54. a. $\cos\frac{3\pi}{4}=-\frac{\sqrt{2}}{2}$

b. $\cos\frac{11\pi}{4}=\cos\left(\frac{3\pi}{4}+2\pi\right)=\cos\frac{3\pi}{4}=-\frac{\sqrt{2}}{2}$

55. a. $\cos\frac{\pi}{2}=0$

b. $\cos\frac{9\pi}{2}=\cos\left(\frac{\pi}{2}+4\pi\right)$
$=\cos\left[\frac{\pi}{2}+2(2\pi)\right]$
$=\cos\frac{\pi}{2}$
$=0$

56. a. $\sin\frac{\pi}{2}=1$

b. $\sin\frac{9\pi}{2}=\sin\left(\frac{\pi}{2}+4\pi\right)=\sin\frac{\pi}{2}=1$

57. a. $\tan\pi=\frac{0}{-1}=0$

b. $\tan 17\pi=\tan(\pi+16\pi)$
$=\tan[\pi+8(2\pi)]$
$=\tan\pi$
$=0$

58. a. $\cot\frac{\pi}{2}=\frac{0}{1}=0$

b. $\cot\frac{15\pi}{2}=\cot\left(\frac{\pi}{2}+7\pi\right)=\cot\frac{\pi}{2}=0$

59. a. $\sin\frac{7\pi}{4}=-\frac{\sqrt{2}}{2}$

b. $\sin\frac{47\pi}{4}=\sin\left(\frac{7\pi}{4}+10\pi\right)$
$=\sin\left[\frac{7\pi}{4}+5(2\pi)\right]$
$=\sin\frac{7\pi}{4}$
$=-\frac{\sqrt{2}}{2}$

60. a. $\cos\frac{7\pi}{4}=\frac{\sqrt{2}}{2}$

b. $\cos\frac{47\pi}{4}=\cos\left(\frac{7\pi}{4}+10\pi\right)=\cos\frac{7\pi}{4}=\frac{\sqrt{2}}{2}$

61. $\sin 0.8\approx 0.7174$

62. $\cos 0.6\approx 0.8253$

63. $\tan 3.4\approx 0.2643$

64. $\tan 3.7\approx 0.6247$

65. $\csc 1\approx 1.1884$

66. $\sec 1\approx 1.8508$

67. $\cos\frac{\pi}{10}\approx 0.9511$

68. $\sin\frac{3\pi}{10}\approx 0.8090$

69. $\cot\frac{\pi}{12}\approx 3.7321$

70. $\cot\frac{\pi}{18}\approx 5.6713$

71. $\sin(-t)-\sin t=-\sin t-\sin t=-2\sin t=-2a$

72. $\tan(-t)-\tan t=-\tan t-\tan t=-2\tan t=-2c$

73. $4\cos(-t)-\cos t = 4\cos t - \cos t = 3\cos t = 3b$

74. $3\cos(-t)-\cos t = 3\cos t - \cos t = 2\cos t = 2b$

75. $\sin(t+2\pi)-\cos(t+4\pi)+\tan(t+\pi)$
$= \sin(t)-\cos(t)+\tan(t)$
$= a-b+c$

76. $\sin(t+2\pi)+\cos(t+4\pi)-\tan(t+\pi)$
$= \sin(t)+\cos(t)-\tan(t)$
$= a+b-c$

77. $\sin(-t-2\pi)-\cos(-t-4\pi)-\tan(-t-\pi)$
$= -\sin(t+2\pi)-\cos(t+4\pi)+\tan(t+\pi)$
$= -\sin(t)-\cos(t)+\tan(t)$
$= -a-b+c$

78. $\sin(-t-2\pi)+\cos(-t-4\pi)-\tan(-t-\pi)$
$= -\sin(t+2\pi)+\cos(t+4\pi)+\tan(t+\pi)$
$= -\sin(t)+\cos(t)+\tan(t)$
$= -a+b+c$

79. $\cos t+\cos(t+1000\pi)-\tan t-\tan(t+999\pi)$
$\quad -\sin t+4\sin(t-1000\pi)$
$= \cos t+\cos t-\tan t-\tan t-\sin t+4\sin t$
$= 2\cos t-2\tan t+3\sin t$
$= 3a+2b-2c$

80. $-\cos t+7\cos(t+1000\pi)+\tan t+\tan(t+999\pi)$
$\quad +\sin t+\sin(t-1000\pi)$
$= -\cos t+7\cos t+\tan t+\tan t+\sin t+\sin t$
$= 6\cos t+2\tan t+2\sin t$
$= 2a+6b+2c$

81. **a.** $H = 12+8.3\sin\left[\frac{2\pi}{365}(80-80)\right]$
$= 12+8.3\sin 0 = 12+8.3(0)$
$= 12$
There are 12 hours of daylight in Fairbanks on March 21.

b. $H = 12+8.3\sin\left[\frac{2\pi}{365}(172-80)\right]$
$\approx 12+8.3\sin 1.5837$
≈ 20.3
There are about 20.3 hours of daylight in Fairbanks on June 21.

c. $H = 12+8.3\sin\left[\frac{2\pi}{365}(355-80)\right]$
$\approx 12+8.3\sin 4.7339$
≈ 3.7
There are about 3.7 hours of daylight in Fairbanks on December 21.

82. **a.** $H = 12+24\sin\left[\frac{2\pi}{365}(80-80)\right]$
$12+24\sin 0 = 12+24(0) = 12$
There are 12 hours of daylight in San Diego on March 21.

b. $H = 12+24\sin\left[\frac{2\pi}{365}(172-80)\right]$
$\approx 12+24\sin 1.5837$
≈ 14.3998
There are about 14.4 hours of daylight in San Diego on June 21.

c. $H = 12+24\sin\left[\frac{2\pi}{365}(355-80)\right]$
$\approx 12+24\sin 4.7339$
≈ 9.6
There are about 9.6 hours of daylight in San Diego on December 21.

83. **a.** For $t = 7$,
$E = \sin\frac{\pi}{14}\cdot 7 = \sin\frac{\pi}{2} = 1$

For $t = 14$,
$E = \sin\frac{\pi}{14}\cdot 14 = \sin\pi = 0$
For $t = 21$,
$E = \sin\frac{\pi}{14}\cdot 21 = \sin\frac{3\pi}{2} = -1$
For $t = 28$,
$E = \sin\frac{\pi}{14}\cdot 28 = \sin 2\pi = \sin 0 = 0$
For $t = 35$,
$E = \sin\frac{\pi}{14}\cdot 35 = \sin\frac{5\pi}{2} = \sin\frac{\pi}{2} = 1$
Observations may vary.

b. Because $E(35) = E(7) = 1$, the period is $35 - 7 = 28$ or 28 days.

84. **a.** At 6 A.M., $t = 0$.

$$H = 10 + 4\sin\frac{\pi}{6}\cdot 0$$
$$= 10 + 4\sin 0 = 10 + 4\cdot 0 = 10$$

The height is 10 feet.

At 9 A.M., $t = 3$.

$$H = 10 + 4\sin\frac{\pi}{6}\cdot 3$$
$$= 10 + 4\sin\frac{\pi}{2} = 10 + 4(1) = 14$$

The height is 14 feet.

At noon, $t = 6$.

$$H = 10 + 4\sin\frac{\pi}{6}\cdot 6$$
$$= 10 + 4\sin\pi = 10 + 4\cdot 0 = 10$$

The height is 10 feet.

At 6 P.M., $t = 12$.

$$H = 10 + 4\sin\frac{\pi}{6}\cdot 12$$
$$= 10 + 4\sin 2\pi = 10 + 4\cdot 0 = 10$$

The height is 10 feet.

At midnight, $t = 18$.

$$H = 10 + 4\sin\frac{\pi}{6}\cdot 18$$
$$= 10 + 4\sin 3\pi = 10 + 4\sin\pi$$
$$= 10 + 4\cdot 0 = 10$$

The height is 10 feet.

At 3 A.M., $t = 21$.

$$H = 10 + 4\sin\frac{\pi}{6}\cdot 21$$
$$= 10 + 4\sin\frac{7\pi}{2} = 10 + 4\sin\frac{3\pi}{2}$$
$$= 10 + 4(-1) = 6$$

The height is 6 feet.

b. The sine function has a minimum at $\frac{3\pi}{2}$. Thus, we find a low tide at $\frac{\pi}{6}t = \frac{3\pi}{2}$ or $t = 9$. This value of t corresponds to 3 P.M. For $t = 9$,

$$h = 10 + 4\sin\frac{\pi}{6}\cdot 9$$
$$= 10 + 4\sin\frac{3\pi}{2} = 10 + 4(-1) = 6$$

The height is 6 feet. From part *a*, the height at 3 A.M. is also 6 feet. Thus, low tide is at 3 A.M. and 3 P.M.

The sine function has a maximum at $\frac{\pi}{2}$. Thus, we find a high tide at $\frac{\pi}{6}t = \frac{\pi}{2}$ or $t = 3$. This value of t corresponds to 9 a.m. From part *a*, the height at 9 A.M. is 14 feet. Because the sine has a period of 2π we also find a maximum at $\frac{5\pi}{2}$. We find another high tide at $\frac{\pi}{6}t = \frac{5\pi}{2}$ or $t = 15$. This value of t corresponds to 9 P.M. Thus, high tide is at 9 A.M. and 9 P.M.

c. The period of the sine function is 2π or on the interval $[0, 2\pi]$. The cycle of the sine function starts at $\frac{\pi}{6}t = \frac{5\pi}{2}$ or $t = 0$, and ends at $\frac{\pi}{6}t = 2\pi$ or $t = 12$. Thus, the period is 12 hours, which means high and low tides occur every 12 hours.

85. – 96. Answers may vary.

97. makes sense

98. does not make sense; Explanations will vary. Sample explanation: $\sin t$ cannot be less than -1. Note that $-\frac{\sqrt{10}}{2} \approx -1.58 < -1$.

99. does not make sense; Explanations will vary. Sample explanation: Cosine is not an odd function.

100. makes sense

101. t is in the third quadrant therefore $\sin t < 0$, $\tan t > 0$, and $\cot t > 0$.
Thus, only choice (c) is true.

102. $f(x) = \sin x$ and $f(a) = \frac{1}{4}$

$$f(a) + f(a + 2\pi) + f(a + 4\pi) + f(a + 6\pi)$$
$$= 4f(a) = 4\left(\frac{1}{4}\right) = 1 \text{ because } \sin x \text{ has a period of } 2\pi.$$

103. $f(x) = \sin x$ and $f(a) = \frac{1}{4}$

$$f(a) + 2f(-a) = f(a) - 2f(a)$$
$$= \frac{1}{4} - 2\left(\frac{1}{4}\right)$$
$$= -\frac{1}{4}$$

$f(-a) = -f(a)$ because $\sin(-x) = -\sin x$.
Sine is an odd function.

104. The height is given by
$h = 45 + 40\sin(t - 90°)$

$$h(765°) = 45 + 40\sin(765° - 90°)$$
$$\approx 16.7$$

You are about 16.7 feet above the ground.

105. First find the hypotenuse.

$$c^2 = a^2 + b^2$$
$$c^2 = 5^2 + 12^2$$
$$c^2 = 25 + 144$$
$$c^2 = 169$$
$$c = 13$$

Next write the ratio.

$$\frac{a}{c} = \frac{5}{13}$$

106. First find the hypotenuse.

$$c^2 = a^2 + b^2$$
$$c^2 = 1^2 + 1^2$$
$$c^2 = 1 + 1$$
$$c^2 = 2$$
$$c = \sqrt{2}$$

Next write the ratio and simplify.

$$\frac{a}{c} = \frac{1}{\sqrt{2}}$$
$$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$
$$= \frac{\sqrt{2}}{2}$$

107. $\left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2 = \frac{a^2}{c^2} + \frac{b^2}{c^2}$

$$= \frac{a^2 + b^2}{c^2}$$

Since $c^2 = a^2 + b^2$, continue simplifying by substituting c^2 for $a^2 + b^2$.

$$\left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2 = \frac{a^2}{c^2} + \frac{b^2}{c^2}$$
$$= \frac{a^2 + b^2}{c^2}$$
$$= \frac{\overbrace{a^2 + b^2}^{c^2}}{c^2}$$
$$= \frac{c^2}{c^2}$$
$$= 1$$

Section 4.3

Checkpoint Exercises

1. Use the Pythagorean Theorem, $c^2 = a^2 + b^2$, to find c.

$$a = 3,\ b = 4$$
$$c^2 = a^2 + b^2 = 3^2 + 4^2 = 9 + 16 = 25$$
$$c = \sqrt{25} = 5$$

Referring to these lengths as opposite, adjacent, and hypotenuse, we have

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{3}{5}$$
$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{4}{5}$$
$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{3}{4}$$
$$\csc\theta = \frac{\text{hypotenuse}}{\text{opposite}} = \frac{5}{3}$$
$$\sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}} = \frac{5}{4}$$
$$\cot\theta = \frac{\text{adjacent}}{\text{opposite}} = \frac{4}{3}$$

2. Use the Pythagorean Theorem, $c^2 = a^2 + b^2$, to find b.

$$a^2 + b^2 = c^2$$
$$1^2 + b^2 = 5^2$$
$$1 + b^2 = 25$$
$$b^2 = 24$$
$$b = \sqrt{24} = 2\sqrt{6}$$

Note that side a is opposite θ and side b is adjacent to θ.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{1}{5}$$
$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{2\sqrt{6}}{5}$$
$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{1}{2\sqrt{6}} = \frac{\sqrt{6}}{12}$$
$$\csc\theta = \frac{\text{hypotenuse}}{\text{opposite}} = \frac{5}{1} = 5$$
$$\sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}} = \frac{5}{2\sqrt{6}} = \frac{5\sqrt{6}}{12}$$
$$\cot\theta = \frac{\text{adjacent}}{\text{opposite}} = \frac{2\sqrt{6}}{1} = 2\sqrt{6}$$

3. Apply the definitions of these three trigonometric functions.

$$\csc 45° = \frac{\text{length of hypotenuse}}{\text{length of side opposite } 45°} = \frac{\sqrt{2}}{1} = \sqrt{2}$$
$$\sec 45° = \frac{\text{length of hypotenuse}}{\text{length of side adjacent to } 45°} = \frac{\sqrt{2}}{1} = \sqrt{2}$$
$$\cot 45° = \frac{\text{length of side adjacent to } 45°}{\text{length of side opposite } 45°} = \frac{1}{1} = 1$$

4.
$$\tan 60° = \frac{\text{length of side opposite } 60°}{\text{length of side adjacent to } 60°} = \frac{\sqrt{3}}{1} = \sqrt{3}$$
$$\tan 30° = \frac{\text{length of side opposite } 30°}{\text{length of side adjacent to } 30°} = \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}}\cdot\frac{\sqrt{3}}{3} = \frac{\sqrt{3}}{3}$$

5. **a.** $\sin 46^{\text{o}} = \cos(90^{\text{o}} - 46^{\text{o}}) = \cos 44^{\text{o}}$

b.
$$\cot\frac{\pi}{12} = \tan\left(\frac{\pi}{2} - \frac{\pi}{12}\right) = \tan\left(\frac{6\pi}{12} - \frac{\pi}{12}\right) = \tan\frac{5\pi}{12}$$

6. Because we have a known angle, an unknown opposite side, and a known adjacent side, we select the
tangent function.

$$\tan 24^{\text{o}} = \frac{a}{750}$$
$$a = 750\tan 24^{\text{o}}$$
$$a \approx 750(0.4452) \approx 333.9$$

The distance across the lake is approximately 333.9 yards.

7. $\tan\theta = \frac{\text{side opposite}}{\text{side adjacent}} = \frac{14}{10}$

Use a calculator in degree mode to find θ.

Many Scientific Calculators	Many Graphing Calculators
$\boxed{\text{TAN}^{-1}}$ $\boxed{(}$ 14 $\boxed{\div}$ 10 $\boxed{)}$ $\boxed{\text{ENTER}}$	$\boxed{\text{TAN}}$ $\boxed{(}$ 14 ÷ 10 $\boxed{)}$ $\boxed{\text{ENTER}}$

The display should show approximately 54. Thus, the angle of elevation of the sun is approximately 54°.

Concept and Vocabulary Check 4.3

1. $\sin\theta = \frac{a}{c}$; $\csc\theta = \frac{c}{a}$; $\cos\theta = \frac{b}{c}$; $\sec\theta = \frac{c}{b}$, $\tan\theta = \frac{a}{b}$; $\cot\theta = \frac{b}{a}$

2. opposite; adjacent to; hypotenuse

3. true

4. $\sin\theta$; $\tan\theta$; $\sec\theta$

Exercise Set 4.3

1. $c^2 = 9^2 + 12^2 = 225$

$c = \sqrt{225} = 15$

$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{9}{15} = \frac{3}{5}$

$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{12}{15} = \frac{4}{5}$

$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{9}{12} = \frac{3}{4}$

$\csc\theta = \frac{\text{hypotenuse}}{\text{opposite}} = \frac{15}{9} = \frac{5}{3}$

$\sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}} = \frac{15}{12} = \frac{5}{4}$

$\cot\theta = \frac{\text{adjacent}}{\text{opposite}} = \frac{12}{9} = \frac{4}{3}$

2. $c^2 = 6^2 + 8^2 = 100$

$c = \sqrt{100} = 10$

$\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}} = \dfrac{6}{10} = \dfrac{3}{5}$

$\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}} = \dfrac{8}{10} = \dfrac{4}{5}$

$\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}} = \dfrac{6}{8} = \dfrac{3}{4}$

$\csc\theta = \dfrac{\text{hypotenuse}}{\text{opposite}} = \dfrac{10}{6} = \dfrac{5}{3}$

$\sec\theta = \dfrac{\text{hypotenuse}}{\text{adjacent}} = \dfrac{10}{8} = \dfrac{5}{4}$

$\cot\theta = \dfrac{\text{adjacent}}{\text{opposite}} = \dfrac{8}{6} = \dfrac{4}{3}$

3. $a^2 + 21^2 = 29^2$

$a^2 = 841 - 441 = 400$

$a = \sqrt{400} = 20$

$\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}} = \dfrac{20}{29}$

$\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}} = \dfrac{21}{29}$

$\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}} = \dfrac{20}{21}$

$\csc\theta = \dfrac{\text{hypotenuse}}{\text{opposite}} = \dfrac{29}{20}$

$\sec\theta = \dfrac{\text{hypotenuse}}{\text{adjacent}} = \dfrac{29}{21}$

$\cot\theta = \dfrac{\text{adjacent}}{\text{opposite}} = \dfrac{21}{20}$

4. $a^2 + 15^2 = 17^2$

$a^2 = 289 - 225 = 64$

$a = \sqrt{64} = 8$

$\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}} = \dfrac{8}{17}$

$\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}} = \dfrac{15}{17}$

$\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}} = \dfrac{8}{15}$

$\csc\theta = \dfrac{\text{hypotenuse}}{\text{opposite}} = \dfrac{17}{8}$

$\sec\theta = \dfrac{\text{hypotenuse}}{\text{adjacent}} = \dfrac{17}{15}$

$\cot\theta = \dfrac{\text{adjacent}}{\text{opposite}} = \dfrac{15}{8}$

5. $10^2 + b^2 = 26^2$

$b^2 = 676 - 100 = 576$

$b = \sqrt{576} = 24$

$\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}} = \dfrac{10}{26} = \dfrac{5}{13}$

$\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}} = \dfrac{24}{26} = \dfrac{12}{13}$

$\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}} = \dfrac{10}{24} = \dfrac{5}{12}$

$\csc\theta = \dfrac{\text{hypotenuse}}{\text{opposite}} = \dfrac{26}{10} = \dfrac{13}{5}$

$\sec\theta = \dfrac{\text{hypotenuse}}{\text{adjacent}} = \dfrac{26}{24} = \dfrac{13}{12}$

$\cot\theta = \dfrac{\text{adjacent}}{\text{opposite}} = \dfrac{24}{10} = \dfrac{12}{5}$

6. $a^2 + 40^2 = 41^2$

$a^2 = 1681 - 1600 = 81$

$a = \sqrt{81} = 9$

$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{9}{41}$

$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{40}{41}$

$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{9}{40}$

$\csc\theta = \frac{\text{hypotenuse}}{\text{opposite}} = \frac{41}{9}$

$\sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}} = \frac{41}{40}$

$\cot\theta = \frac{\text{adjacent}}{\text{opposite}} = \frac{40}{9}$

7. $21^2 + b^2 = 35^2$

$b^2 = 1225 - 441 = 784$

$b = \sqrt{784} = 28$

$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{28}{35} = \frac{4}{5}$

$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{21}{35} = \frac{3}{5}$

$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{28}{21} = \frac{4}{3}$

$\csc\theta = \frac{\text{hypotenuse}}{\text{opposite}} = \frac{35}{28} = \frac{5}{4}$

$\sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}} = \frac{35}{21} = \frac{5}{3}$

$\cot\theta = \frac{\text{adjacent}}{\text{opposite}} = \frac{21}{28} = \frac{3}{4}$

8. $a^2 + 24^2 = 25^2$

$a^2 = 625 - 576 = 49$

$a = \sqrt{49} = 7$

$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{24}{25}$

$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{7}{25}$

$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{24}{7}$

$\csc\theta = \frac{\text{hypotenuse}}{\text{opposite}} = \frac{25}{24}$

$\sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}} = \frac{25}{7}$

$\cot\theta = \frac{\text{adjacent}}{\text{opposite}} = \frac{7}{24}$

9. $\cos 30^\circ = \frac{\text{length of side adjacent to } 30^\circ}{\text{length of hypotenuse}}$

$= \frac{\sqrt{3}}{2}$

10. $\tan 30^\circ = \frac{\text{length of side opposite } 30^\circ}{\text{length of side adjacent to } 30^\circ}$

$= \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

11. $\sec 45^\circ = \frac{\text{length of hypotenuse}}{\text{length of side adjacent to } 45^\circ}$

$= \frac{\sqrt{2}}{1} = \sqrt{2}$

12. $\csc 45^\circ = \frac{\text{length of hypotenuse}}{\text{length of side opposite } 45^\circ}$

$= \frac{\sqrt{2}}{1} = \sqrt{2}$

13. $\tan\frac{\pi}{3} = \tan 60^\circ$

$= \frac{\text{length of side opposite } 60^\circ}{\text{length of side adjacent to } 60^\circ}$

$= \frac{\sqrt{3}}{1} = \sqrt{3}$

14. $\cot\frac{\pi}{3} = \cot 60° = \frac{\text{length of side adjacent to } 60°}{\text{length of side opposite } 60°}$

$= \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

15. $\sin\frac{\pi}{4} - \cos\frac{\pi}{4} = \sin 45° - \cos 45°$

$= \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = 0$

16. $\tan\frac{\pi}{4} + \csc\frac{\pi}{6} = \tan 45° + \csc 30°$

$= \frac{1}{1} + \frac{2}{1} = 1 + 2 = 3$

17. $\sin\frac{\pi}{3}\cos\frac{\pi}{4} - \tan\frac{\pi}{4} = \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - 1$

$= \frac{\sqrt{6}}{4} - 1$

$= \frac{\sqrt{6} - 4}{4}$

18. $\cos\frac{\pi}{3}\sec\frac{\pi}{3} - \cot\frac{\pi}{3} = 1 - \frac{\sqrt{3}}{3} = \frac{3 - \sqrt{3}}{3}$

19. $2\tan\frac{\pi}{3} + \cos\frac{\pi}{4}\tan\frac{\pi}{6} = 2\left(\sqrt{3}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{3}\right)$

$= 2\sqrt{3} + \frac{\sqrt{6}}{6}$

$= \frac{12\sqrt{3} + \sqrt{6}}{6}$

20. $6\tan\frac{\pi}{4} + \sin\frac{\pi}{3}\sec\frac{\pi}{6} = 6(1) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{2\sqrt{3}}{3}\right)$

$= 6 + \frac{6}{6}$

$= 7$

21. $\sin 7° = \cos(90° - 7°) = \cos 83°$

22. $\sin 19° = \cos\left(90° - 19°\right) = \cos 71°$

23. $\csc 25° = \sec(90° - 25°) = \sec 65°$

24. $\csc 35° = \sec(90° - 35°) = \sec 55°$

25. $\tan\frac{\pi}{9} = \cot\left(\frac{\pi}{2} - \frac{\pi}{9}\right)$

$= \cot\left(\frac{9\pi}{18} - \frac{2\pi}{18}\right)$

$= \cot\frac{7\pi}{18}$

26. $\tan\frac{\pi}{7} = \cot\left(\frac{\pi}{2} - \frac{\pi}{7}\right) = \cot\left(\frac{7\pi}{14} - \frac{2\pi}{14}\right) = \cot\frac{5\pi}{14}$

27. $\cos\frac{2\pi}{5} = \sin\left(\frac{\pi}{2} - \frac{2\pi}{5}\right)$

$= \sin\left(\frac{5\pi}{10} - \frac{4\pi}{10}\right)$

$= \sin\frac{\pi}{10}$

28. $\cos\frac{3\pi}{8} = \sin\left(\frac{\pi}{2} - \frac{3\pi}{8}\right) = \sin\left(\frac{4\pi}{8} - \frac{3\pi}{8}\right) = \sin\frac{\pi}{8}$

29. $\tan 37° = \frac{a}{250}$

$a = 250\tan 37°$

$a \approx 250(0.7536) \approx 188 \text{ cm}$

30. $\tan 61° = \frac{a}{10}$

$a = 10\tan 61°$

$a \approx 10(1.8040) \approx 18 \text{ cm}$

31. $\cos 34° = \frac{b}{220}$

$b = 220\cos 34°$

$b \approx 220(0.8290) \approx 182 \text{ in.}$

32. $\sin 34° = \frac{a}{13}$

$a = 13\sin 34°$

$a \approx 13(0.5592) \approx 7 \text{ m}$

33. $\sin 23° = \frac{16}{c}$

$c = \frac{16}{\sin 23°} \approx \frac{16}{0.3907} \approx 41 \text{ m}$

34. $\tan 44° = \dfrac{23}{b}$

$b = \dfrac{23}{\tan 44°} \approx \dfrac{23}{0.9657} \approx 24 \text{ yd}$

35.

Scientific Calculator	Graphing Calculator	Display (rounded to the nearest degree)
0.2974 $\boxed{\text{SIN}^{-1}}$	$\boxed{\text{SIN}^{-1}}$ 0.2974 $\boxed{\text{ENTER}}$	17

If $\sin\theta = 0.2974$, then $\theta \approx 17°$.

36.

Scientific Calculator	Graphing Calculator	Display (rounded to the nearest degree)
0.877 $\boxed{\text{COS}^{-1}}$	$\boxed{\text{COS}^{-1}}$ 0.877 $\boxed{\text{ENTER}}$	29

If $\cos\theta = 0.877$, then $\theta \approx 29°$.

37.

Scientific Calculator	Graphing Calculator	Display (rounded to the nearest degree)
4.6252 $\boxed{\text{TAN}^{-1}}$	$\boxed{\text{TAN}^{-1}}$ 4.6252 $\boxed{\text{ENTER}}$	78

If $\tan\theta = 4.6252$, then $\theta \approx 78°$.

38.

Scientific Calculator	Graphing Calculator	Display (rounded to the nearest degree)
26.0307 $\boxed{\text{TAN}^{-1}}$	$\boxed{\text{TAN}^{-1}}$ 26.0307 $\boxed{\text{ENTER}}$	88

If $\tan\theta = 26.0307$, then $\theta \approx 88°$.

39.

Scientific Calculator	Graphing Calculator	Display (rounded to three places)
0.4112 $\boxed{\text{COS}^{-1}}$	$\boxed{\text{COS}^{-1}}$ 0.4112 $\boxed{\text{ENTER}}$	1.147

If $\cos\theta = 0.4112$, then $\theta \approx 1.147$ radians.

40.

Scientific Calculator	Graphing Calculator	Display (rounded to three places)
0.9499 $\boxed{\text{SIN}^{-1}}$	$\boxed{\text{SIN}^{-1}}$ 0.9499 $\boxed{\text{ENTER}}$	1.253

If $\sin\theta = 0.9499$, then $\theta = 1.253$ radians.

41.

Scientific Calculator	Graphing Calculator	Display (rounded to three places)
0.4169 $\boxed{\text{TAN}^{-1}}$	$\boxed{\text{TAN}^{-1}}$ 0.4169 $\boxed{\text{ENTER}}$	0.395

If $\tan\theta = 0.4169$, then $\theta \approx 0.395$ radians.

42.

Scientific Calculator	Graphing Calculator	Display (rounded to three places)
0.5117 $\boxed{\text{TAN}^{-1}}$	$\boxed{\text{TAN}^{-1}}$ 0.5117 $\boxed{\text{ENTER}}$	0.473

If $\tan\theta = 0.5117$, then $\theta = 0.473$

43. $\dfrac{\tan\frac{\pi}{3}}{2} - \dfrac{1}{\sec\frac{\pi}{6}} = \dfrac{\sqrt{3}}{2} - \dfrac{1}{\frac{1}{\cos\frac{\pi}{6}}}$

$= \dfrac{\sqrt{3}}{2} - \dfrac{1}{\frac{1}{\frac{\sqrt{3}}{2}}}$

$= \dfrac{\sqrt{3}}{2} - \dfrac{\sqrt{3}}{2}$

$= 0$

44. $\dfrac{1}{\cot\frac{\pi}{4}} - \dfrac{2}{\csc\frac{\pi}{6}} = \dfrac{1}{\frac{1}{\tan\frac{\pi}{4}}} - \dfrac{2}{\frac{1}{\sin\frac{\pi}{6}}}$

$= \dfrac{1}{\frac{1}{1}} - \dfrac{2}{\frac{1}{1/2}}$

$= \dfrac{1}{1} - \dfrac{2}{2}$

$= 1 - 1$

$= 0$

45. $1 + \sin^2 40° + \sin^2 50°$

$= 1 + \sin^2(90° - 50°) + \sin^2 50°$

$= 1 + \cos^2 50° + \sin^2 50°$

$= 1 + 1$

$= 2$

46. $1 - \tan^2 10° + \csc^2 80°$

$= 1 - \cot^2 80° + \csc^2 80°$

$= 1 + \csc^2 80° - \cot^2 80°$

$= 1 + 1$

$= 2$

47. $\csc 37° \sec 53° - \tan 53° \cot 37°$

$= \sec 53° \sec 53° - \tan 53° \tan 53°$

$= \sec^2 53° - \tan^2 53°$

$= 1$

48. $\cos 12° \sin 78° + \cos 78° \sin 12°$

$= \sin 78° \sin 78° + \cos 78° \cos 78°$

$= \sin^2 78° + \cos^2 78°$

$= 1$

49. $f(\theta) = 2\cos\theta - \cos 2\theta$

$f\left(\dfrac{\pi}{6}\right) = 2\cos\dfrac{\pi}{6} - \cos\left(2 \cdot \dfrac{\pi}{6}\right)$

$= 2\left(\dfrac{\sqrt{3}}{2}\right) - \cos\left(\dfrac{\pi}{3}\right)$

$= \dfrac{2\sqrt{3}}{2} - \dfrac{1}{2}$

$= \dfrac{2\sqrt{3} - 1}{2}$

50. $f(\theta) = 2\sin\theta - \sin\dfrac{\theta}{2}$

$f\left(\dfrac{\pi}{3}\right) = 2\sin\dfrac{\pi}{3} - \sin\dfrac{\frac{\pi}{3}}{2}$

$= 2\left(\dfrac{\sqrt{3}}{2}\right) - \sin\left(\dfrac{\pi}{6}\right)$

$= \dfrac{2\sqrt{3}}{2} - \dfrac{1}{2}$

$= \dfrac{2\sqrt{3} - 1}{2}$

51. $\tan\left(\dfrac{\pi}{2} - \theta\right) = \cot\theta = \dfrac{1}{4}$

52. $\csc\left(\dfrac{\pi}{2} - \theta\right) = \sec\theta = \dfrac{1}{\cos\theta} = \dfrac{1}{\frac{1}{3}} = 3$

53. $\tan 40° = \dfrac{a}{630}$

$a = 630 \tan 40°$

$a \approx 630(0.8391) \approx 529$

The distance across the lake is approximately 529 yards.

54. $\tan 40° = \dfrac{h}{35}$

$h = 35 \tan 40°$

$h \approx 35(0.8391) \approx 29$

The tree's height is approximately 29 feet.

55. $\tan\theta = \dfrac{125}{172}$

Use a calculator in degree mode to find θ.

Many Scientific Calculators	Many Graphing Calculators
125 $\boxed{\div}$ 172 $\boxed{=}$ $\boxed{\text{TAN}^{-1}}$	$\boxed{\text{TAN}^{-1}}$ $\boxed{(}$ 125 $\boxed{\div}$ 172 $\boxed{)}$ $\boxed{\text{ENTER}}$

The display should show approximately 36. Thus, the angle of elevation of the sun is approximately 36°.

56. $\tan\dfrac{555}{1320}$

Use a calculator in degree mode to find θ.

Many Scientific Calculators	Many Graphing Calculators
555 ÷ 1320 $\boxed{=}$ $\boxed{\text{TAN}^{-1}}$	$\boxed{\text{TAN}^{-1}}$ $\boxed{(}$ 555 ÷ 1320 $\boxed{)}$ $\boxed{\text{ENTER}}$

The display should show approximately 23. Thus, the angle of elevation is approximately 23°.

57. $\sin 10° = \dfrac{500}{c}$

$c = \dfrac{500}{\sin 10°} \approx \dfrac{500}{0.1736} \approx 2880$

The plane has flown approximately 2880 feet.

58. $\sin 5° = \dfrac{a}{5000}$

$a = 5000 \sin 5° \approx 5000(0.0872) = 436$

The driver's increase in altitude was approximately 436 feet.

59. $\cos\theta = \dfrac{60}{75}$

Use a calculator in degree mode to find θ.

Many Scientific Calculators	Many Graphing Calculators
60 $\boxed{\div}$ 75 $\boxed{=}$ $\boxed{\text{COS}^{-1}}$	$\boxed{\text{COS}^{-1}}$ $\boxed{(}$ 60 $\boxed{\div}$ 75 $\boxed{)}$ $\boxed{\text{ENTER}}$

The display should show approximately 37. Thus, the angle between the wire and the pole is approximately 37°.

60. $\cos\theta = \dfrac{55}{80}$

Use a calculator in degree mode to find θ.

Many Scientific Calculators	Many Graphing Calculators
55 ÷ 80 $\boxed{=}$ $\boxed{\text{COS}^{-1}}$	$\boxed{\text{COS}^{-1}}$ $\boxed{(}$ 55 ÷ 80 $\boxed{)}$ $\boxed{\text{ENTER}}$

The display should show approximately 47. Thus, the angle between the wire and the pole is approximately 47°.

61. – 67. Answers may vary.

68.

θ	0.4	0.3	0.2	0.1	0.01	0.001	0.0001	0.00001
$\sin\theta$	0.3894	0.2955	0.1987	0.0998	0.0099998	9.999998×10^{-4}	9.99999998×10^{-5}	1×10^{-5}
$\frac{\sin\theta}{\theta}$	0.9736	0.9851	0.9933	0.9983	0.99998	0.9999998	0.999999998	1

$\frac{\sin\theta}{\theta}$ approaches 1 as θ approaches 0.

69.

θ	0.4	0.3	0.2	0.1	0.01	0.001	0.0001	0.00001
$\cos\theta$	0.92106	0.95534	0.98007	0.99500	0.99995	0.9999995	0.999999995	1
$\frac{\cos\theta-1}{\theta}$	–0.19735	–0.148878	–0.099667	–0.04996	–0.005	–0.0005	–0.00005	0

$\frac{\cos\theta-1}{\theta}$ approaches 0 as θ approaches 0.

70. does not make sense; Explanations will vary. Sample explanation: An increase in the size of a triangle does not affect the ratios of the sides.

71. does not make sense; Explanations will vary. Sample explanation: This value is irrational. Irrational numbers are rounded on calculators.

72. does not make sense; Explanations will vary. Sample explanation: The sine and cosine functions are not reciprocal functions of each other.

73. makes sense

74. false; Changes to make the statement true will vary. A sample change is: $\frac{\tan 45°}{\tan 15°} \neq \tan\left(\frac{45°}{15°}\right)$

75. true

76. false; Changes to make the statement true will vary. A sample change is: $\sin 45° + \cos 45° = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} \neq 1$

77. true

78. In a right triangle, the hypotenuse is greater than either other side. Therefore both $\frac{\text{opposite}}{\text{hypotenuse}}$ and $\frac{\text{adjacent}}{\text{hypotenuse}}$ must be less than 1 for an acute angle in a right triangle.

79. Use a calculator in degree mode to generate the following table. Then use the table to describe what happens to the tangent of an acute angle as the angle gets close to 90°.

θ	60	70	80	89	89.9	89.99	89.999	89.9999
$\tan\theta$	1.7321	2.7475	5.6713	57	573	5730	57,296	572,958

As θ approaches 90°, $\tan\theta$ increases without bound. At 90°, $\tan\theta$ is undefined.

80. **a.** Let a = distance of the ship from the lighthouse.

$$\tan 35° = \frac{250}{a}$$

$$a = \frac{250}{\tan 35°} \approx \frac{250}{0.7002} \approx 357$$

The ship is approximately 357 feet from the lighthouse.

b. Let b = the plane's height above the lighthouse.

$$\tan 22° = \frac{b}{357}$$

$$b = 357\tan 22° \approx 357(0.4040) \approx 144$$

$$144 + 250 = 394$$

The plane is approximately 394 feet above the water.

81. **a.** $\frac{y}{r}$

b. First find r: $r = \sqrt{x^2 + y^2}$

$$r = \sqrt{(-3)^2 + 4^2}$$

$$r = 5$$

$\frac{y}{r} = \frac{4}{5}$, which is positive.

82. **a.** $\frac{x}{r}$

b. First find r: $r = \sqrt{x^2 + y^2}$

$$r = \sqrt{(-3)^2 + 5^2}$$

$$r = \sqrt{34}$$

$\frac{x}{r} = \frac{-3}{\sqrt{34}} = \frac{-3}{\sqrt{34}} \cdot \frac{\sqrt{34}}{\sqrt{34}} = \frac{-3\sqrt{34}}{34}$, which is negative.

83. **a.** $\theta' = 360^{\circ} - 345^{\circ} = 15^{\circ}$

b. $\theta' = \pi - \frac{5\pi}{6} = \frac{6\pi}{6} - \frac{5\pi}{6} = \frac{\pi}{6}$

Section 4.4

Checkpoint Exercises

1. $r = \sqrt{x^2 + y^2}$

$$r = \sqrt{1^2 + (-3)^2} = \sqrt{1+9} = \sqrt{10}$$

Now that we know x, y, and r, we can find the six trigonometric functions of θ.

$$\sin\theta = \frac{y}{r} = \frac{-3}{\sqrt{10}} = -\frac{3\sqrt{10}}{10}$$

$$\cos\theta = \frac{x}{r} = \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10}$$

$$\tan\theta = \frac{y}{x} = \frac{-3}{1} = -3$$

$$\csc\theta = \frac{r}{y} = \frac{\sqrt{10}}{-3} = -\frac{\sqrt{10}}{3}$$

$$\sec\theta = \frac{r}{x} = \frac{\sqrt{10}}{1} = \sqrt{10}$$

$$\cot\theta = \frac{x}{y} = \frac{1}{-3} = -\frac{1}{3}$$

2. **a.** $\theta = 0° = 0$ radians

The terminal side of the angle is on the positive x-axis. Select the point $P = (1,0)$: $x = 1, y = 0, r = 1$

Apply the definitions of the cosine and cosecant functions.

$$\cos 0° = \cos 0 = \frac{x}{r} = \frac{1}{1} = 1$$

$$\csc 0° = \csc 0 = \frac{r}{y} = \frac{1}{0}, \text{ undefined}$$

b. $\theta = 90° = \frac{\pi}{2}$ radians

The terminal side of the angle is on the positive y-axis. Select the point $P = (0,1)$: $x = 0, y = 1, r = 1$

Apply the definitions of the cosine and cosecant functions.

$$\cos 90° = \cos\frac{\pi}{2} = \frac{x}{r} = \frac{0}{1} = 0$$

$$\csc 90° = \csc\frac{\pi}{2} = \frac{r}{y} = \frac{1}{1} = 1$$

c. $\theta = 180° = \pi$ radians
The terminal side of the angle is on the negative x-axis. Select the point
$P = (-1,0)$: $x = -1, y = 0, r = 1$
Apply the definitions of the cosine and cosecant functions.

$$\cos 180° = \cos \pi = \frac{x}{r} = \frac{-1}{1} = -1$$

$$\csc 180° = \csc \pi = \frac{r}{y} = \frac{1}{0}, \text{ undefined}$$

d. $\theta = 270° = \frac{3\pi}{2}$ radians
The terminal side of the angle is on the negative y-axis. Select the point
$P = (0,-1)$: $x = 0, y = -1, r = 1$
Apply the definitions of the cosine and cosecant functions.

$$\cos 270° = \cos \frac{3\pi}{2} = \frac{x}{r} = \frac{0}{1} = 0$$

$$\csc 270° = \csc \frac{3\pi}{2} = \frac{r}{y} = \frac{1}{-1} = -1$$

3. Because $\sin\theta < 0$, θ cannot lie in quadrant I; all the functions are positive in quadrant I. Furthermore, θ cannot lie in quadrant II; $\sin\theta$ is positive in quadrant II. Thus, with $\sin\theta < 0$, θ lies in quadrant III or quadrant IV. We are also given that $\cos\theta < 0$. Because quadrant III is the only quadrant in which cosine is negative and the sine is negative, we conclude that θ lies in quadrant III.

4. Because the tangent is negative and the cosine is negative, θ lies in quadrant II. In quadrant II, x is negative and y is positive. Thus,

$$\tan\theta = -\frac{1}{3} = \frac{y}{x} = \frac{1}{-3}$$

$x = -3,\ y = 1$

Furthermore,

$$r = \sqrt{x^2 + y^2} = \sqrt{(-3)^2 + 1^2} = \sqrt{9+1} = \sqrt{10}$$

Now that we know x, y, and r, we can find $\sin\theta$ and $\sec\theta$.

$$\sin\theta = \frac{y}{r} = \frac{1}{\sqrt{10}} = \frac{1}{\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}} = \frac{\sqrt{10}}{10}$$

$$\sec\theta = \frac{r}{x} = \frac{\sqrt{10}}{-3} = -\frac{\sqrt{10}}{3}$$

5. a. Because 210° lies between 180° and 270°, it is in quadrant III. The reference angle is $\theta' = 210° - 180° = 30°$.

b. Because $\frac{7\pi}{4}$ lies between $\frac{3\pi}{2} = \frac{6\pi}{4}$ and $2\pi = \frac{8\pi}{4}$, it is in quadrant IV. The reference angle is $\theta' = 2\pi - \frac{7\pi}{4} = \frac{8\pi}{4} - \frac{7\pi}{4} = \frac{\pi}{4}$.

c. Because –240° lies between –180° and –270°, it is in quadrant II. The reference angle is $\theta = 240 - 180 = 60°$.

d. Because 3.6 lies between $\pi \approx 3.14$ and $\frac{3\pi}{2} \approx 4.71$, it is in quadrant III. The reference angle is $\theta' = 3.6 - \pi \approx 0.46$.

6. a. $665° - 360° = 305°$
This angle is in quadrant IV, thus the reference angle is $\theta' = 360° - 305° = 55°$.

b. $\frac{15\pi}{4} - 2\pi = \frac{15\pi}{4} - \frac{8\pi}{4} = \frac{7\pi}{4}$
This angle is in quadrant IV, thus the reference angle is $\theta' = 2\pi - \frac{7\pi}{4} = \frac{8\pi}{4} - \frac{7\pi}{4} = \frac{\pi}{4}$.

c. $-\frac{11\pi}{3} + 2 \cdot 2\pi = -\frac{11\pi}{3} + \frac{12\pi}{3} = \frac{\pi}{3}$
This angle is in quadrant I, thus the reference angle is $\theta' = \frac{\pi}{3}$.

7. a. 300° lies in quadrant IV. The reference angle is $\theta' = 360° - 300° = 60°$.

$$\sin 60° = \frac{\sqrt{3}}{2}$$

Because the sine is negative in quadrant IV,

$$\sin 300° = -\sin 60° = -\frac{\sqrt{3}}{2}.$$

b. $\frac{5\pi}{4}$ lies in quadrant III. The reference angle is

$$\theta' = \frac{5\pi}{4} - \pi = \frac{5\pi}{4} - \frac{4\pi}{4} = \frac{\pi}{4}.$$

$$\tan\frac{\pi}{4} = 1$$

Because the tangent is positive in quadrant III,

$$\tan\frac{5\pi}{4} = +\tan\frac{\pi}{4} = 1.$$

c. $-\frac{\pi}{6}$ lies in quadrant IV. The reference angle is $\theta' = \frac{\pi}{6}$.

$\sec\frac{\pi}{6} = \frac{2\sqrt{3}}{3}$

Because the secant is positive in quadrant IV,

$\sec\left(-\frac{\pi}{6}\right) = +\sec\frac{\pi}{6} = \frac{2\sqrt{3}}{3}$.

8. a. $\frac{17\pi}{6} - 2\pi = \frac{17\pi}{6} - \frac{12\pi}{6} = \frac{5\pi}{6}$ lies in quadrant II. The reference angle is $\theta' = \pi - \frac{5\pi}{6} = \frac{\pi}{6}$.

The function value for the reference angle is $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$.

Because the cosine is negative in quadrant II,

$\cos\frac{17\pi}{6} = \cos\frac{5\pi}{6} = -\cos\frac{\pi}{6} = -\frac{\sqrt{3}}{2}$.

b. $\frac{-22\pi}{3} + 8\pi = \frac{-22\pi}{3} + \frac{24\pi}{3} = \frac{2\pi}{3}$ lies in quadrant II. The reference angle is $\theta' = \pi - \frac{2\pi}{3} = \frac{\pi}{3}$.

The function value for the reference angle is $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$.

Because the sine is positive in quadrant II,

$\sin\frac{-22\pi}{3} = \sin\frac{2\pi}{3} = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$.

Concept and Vocabulary Check 4.4

1. $\sin\theta = \frac{y}{r}$; $\csc\theta = \frac{r}{y}$; $\cos\theta = \frac{x}{r}$; $\sec\theta = \frac{r}{x}$'

$\tan\theta = \frac{y}{x}$; $\cot\theta = \frac{x}{y}$

2. $\tan\theta$; $\sec\theta$; $\cot\theta$; $\csc\theta$; $\tan\theta$; $\cot\theta$

3. $\sin\theta$; $\csc\theta$;

4. $\tan\theta$; $\cot\theta$;

5. $\cos\theta$; $\sec\theta$

6. terminal; x

7. **(a)** $180° - \theta$; **(b)** $\theta - 180°$; **(c)** $360° - \theta$

Exercise Set 4.4

1. We need values for x, y, and r. Because $P = (-4, 3)$ is a point on the terminal side of θ, $x = -4$ and $y = 3$. Furthermore,

$r = \sqrt{x^2 + y^2} = \sqrt{(-4)^2 + 3^2} = \sqrt{16+9} = \sqrt{25} = 5$ No w that we know x, y, and r, we can find the six trigonometric functions of θ.

$\sin\theta = \frac{y}{r} = \frac{3}{5}$

$\cos\theta = \frac{x}{r} = \frac{-4}{5} = -\frac{4}{5}$

$\tan\theta = \frac{y}{x} = \frac{3}{-4} = -\frac{3}{4}$

$\csc\theta = \frac{r}{y} = \frac{5}{3}$

$\sec\theta = \frac{r}{x} = \frac{5}{-4} = -\frac{5}{4}$

$\cot\theta = \frac{x}{y} = \frac{-4}{3} = -\frac{4}{3}$

2. We need values for x, y, and r, Because $P = (-12, 5)$ is a point on the terminal side of θ, $x = -12$ and $y = 5$. Furthermore,

$r = \sqrt{x^2 + y^2} = \sqrt{(-12)^2 + 5^2} = \sqrt{144+25}$

$= \sqrt{169} = 13$

Now that we know x, y, and r, we can find the six trigonometric functions of θ.

$\sin\theta = \frac{y}{r} = \frac{5}{13}$

$\cos\theta = \frac{x}{r} = \frac{-12}{13} = -\frac{12}{13}$

$\tan\theta = \frac{y}{x} = \frac{5}{-12} = -\frac{5}{12}$

$\csc\theta = \frac{r}{y} = \frac{13}{5}$

$\sec\theta = \frac{r}{x} = \frac{13}{-12} = -\frac{13}{12}$

$\cot\theta = \frac{x}{y} = \frac{-12}{5} - \frac{12}{5}$

3. We need values for *x, y,* and *r*. Because $P = (2, 3)$ is a point on the terminal side of θ, $x = 2$ and $y = 3$. Furthermore,

$$r = \sqrt{x^2 + y^2} = \sqrt{2^2 + 3^2} = \sqrt{4+9} = \sqrt{13}$$

Now that we know *x, y,* and *r,* we can find the six trigonometric functions of θ.

$$\sin\theta = \frac{y}{r} = \frac{3}{\sqrt{13}} = \frac{3}{\sqrt{13}} \cdot \frac{\sqrt{13}}{\sqrt{13}} = \frac{3\sqrt{13}}{13}$$

$$\cos\theta = \frac{x}{r} = \frac{2}{\sqrt{13}} = \frac{2}{\sqrt{13}} \cdot \frac{\sqrt{13}}{\sqrt{13}} = \frac{2\sqrt{13}}{13}$$

$$\tan\theta = \frac{y}{x} = \frac{3}{2}$$

$$\csc\theta = \frac{r}{y} = \frac{\sqrt{13}}{3}$$

$$\sec\theta = \frac{r}{x} = \frac{\sqrt{13}}{2}$$

$$\cot\theta = \frac{x}{y} = \frac{2}{3}$$

4. We need values for *x, y,* and *r,* Because $P = (3, 7)$ is a point on the terminal side of θ, $x = 3$ and $y = 7$. Furthermore,

$$r = \sqrt{x^2 + y^2} = \sqrt{3^2 + 7^2} = \sqrt{9+49} = \sqrt{58}$$

Now that we know *x, y,* and *r,* we can find the six trigonometric functions of θ.

$$\sin\theta = \frac{y}{r} = \frac{7}{\sqrt{58}} = \frac{7}{\sqrt{58}} \cdot \frac{\sqrt{58}}{\sqrt{58}} = \frac{7\sqrt{58}}{58}$$

$$\cos\theta = \frac{x}{r} = \frac{3}{\sqrt{58}} = \frac{3}{\sqrt{58}} \cdot \frac{\sqrt{58}}{\sqrt{58}} = \frac{3\sqrt{58}}{58}$$

$$\tan\theta = \frac{y}{x} = \frac{7}{3}$$

$$\csc\theta = \frac{r}{y} = \frac{\sqrt{58}}{7}$$

$$\sec\theta = \frac{r}{x} = \frac{\sqrt{58}}{3}$$

$$\cot\theta = \frac{x}{y} = \frac{3}{7}$$

5. We need values for *x, y,* and *r*. Because $P = (3, -3)$ is a point on the terminal side of θ, $x = 3$ and $y = -3$.

Furthermore, $r = \sqrt{x^2 + y^2} = \sqrt{3^2 + (-3)^2} = \sqrt{9+9}$
$= \sqrt{18} = 3\sqrt{2}$

Now that we know *x, y,* and *r,* we can find the six trigonometric functions of θ.

$$\sin\theta = \frac{y}{r} = \frac{-3}{3\sqrt{2}} = -\frac{-1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$$

$$\cos\theta = \frac{x}{r} = \frac{3}{3\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\tan\theta = \frac{y}{x} = \frac{-3}{3} = -1$$

$$\csc\theta = \frac{r}{y} = \frac{3\sqrt{2}}{-3} = -\sqrt{2}$$

$$\sec\theta = \frac{r}{x} = \frac{3\sqrt{2}}{3} = \sqrt{2}$$

$$\cot\theta = \frac{x}{y} = \frac{3}{-3} = -1$$

6. We need values for *x, y,* and *r,* Because $P = (5, -5)$ is a point on the terminal side of θ, $x = 5$ and $y = -5$. Furthermore,

$$r = \sqrt{x^2 + y^2} = \sqrt{5 + (-5)^2} = \sqrt{25+25} = \sqrt{50}$$
$$= 5\sqrt{2}$$

Now that we know *x, y,* and *r,* we can find the six trigonometric functions of θ.

$$\sin\theta = \frac{y}{r} = \frac{-5}{5\sqrt{2}} = \frac{-1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$$

$$\cos\theta = \frac{x}{r} = \frac{5}{5\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\tan\theta = \frac{y}{x} = \frac{-5}{5} = -1$$

$$\csc\theta = \frac{r}{y} = \frac{5\sqrt{2}}{-5} = -\sqrt{2}$$

$$\sec\theta = \frac{r}{x} = \frac{5\sqrt{2}}{5} = \sqrt{2}$$

$$\cot\theta = \frac{x}{y} = \frac{5}{-5} = -1$$

7. We need values for *x, y,* and *r*. Because $P = (-2, -5)$ is a point on the terminal side of θ, $x = -2$ and $y = -5$. Furthermore,

$r = \sqrt{x^2 + y^2} = \sqrt{(-2)^2 + (-5)^2} = \sqrt{4+25} = \sqrt{29}$ No

w that we know *x, y,* and *r,* we can find the six trigonometric functions of θ.

$$\sin\theta = \frac{y}{r} = \frac{-5}{\sqrt{29}} = \frac{-5}{\sqrt{29}} \cdot \frac{\sqrt{29}}{\sqrt{29}} = -\frac{5\sqrt{29}}{29}$$

$$\cos\theta = \frac{x}{r} = \frac{-2}{\sqrt{29}} = \frac{-2}{\sqrt{29}} \cdot \frac{\sqrt{29}}{\sqrt{29}} = -\frac{2\sqrt{29}}{29}$$

$$\tan\theta = \frac{y}{x} = \frac{-5}{-2} = \frac{5}{2}$$

$$\csc\theta = \frac{r}{y} = \frac{\sqrt{29}}{-5} = -\frac{\sqrt{29}}{5}$$

$$\sec\theta = \frac{r}{x} = \frac{\sqrt{29}}{-2} = -\frac{\sqrt{29}}{2}$$

$$\cot\theta = \frac{x}{y} = \frac{-2}{-5} = \frac{2}{5}$$

8. We need values for *x, y,* and *r,* Because $P = (-1, -3)$ is a point on the terminal side of θ, $x = -1$ and $y = -3$. Furthermore,

$r = \sqrt{x^2 + y^2} = \sqrt{(-1)^2 + (-3)^2} = \sqrt{1+9} = \sqrt{10}$

Now that we know *x, y,* and *r,* we can find the six trigonometric functions of θ.

$$\sin\theta = \frac{y}{r} = \frac{-3}{\sqrt{10}} = \frac{-3}{\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}} = -\frac{3\sqrt{10}}{10}$$

$$\cos\theta = \frac{x}{r} = \frac{-1}{\sqrt{10}} = \frac{-1}{\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}} = -\frac{\sqrt{10}}{10}$$

$$\tan\theta = \frac{y}{x} = \frac{-3}{-1} = 3$$

$$\csc\theta = \frac{r}{y} = \frac{\sqrt{10}}{-3} = -\frac{\sqrt{10}}{3}$$

$$\sec\theta = \frac{r}{x} = \frac{\sqrt{10}}{-1} = -\sqrt{10}$$

$$\cot\theta = \frac{x}{y} = \frac{-1}{-3} = \frac{1}{3}$$

9. $\theta = \pi$ radians

The terminal side of the angle is on the negative *x*-axis. Select the point $P = (-1, 0)$: $x = -1,\ y = 0,\ r = 1$ Apply the definition of the cosine function.

$$\cos\pi = \frac{x}{r} = \frac{-1}{1} = -1$$

10. $\theta = \pi$ radians

The terminal side of the angle is on the negative *x*-axis. Select the point $P = (-1, 0)$: $x = -1, y = 0,\ r = 1$

Apply the definition of the tangent function.

$$\tan\pi = \frac{y}{x} = \frac{0}{-1} = 0$$

11. $\theta = \pi$ radians

The terminal side of the angle is on the negative *x*-axis. Select the point $P = (-1, 0)$: $x = -1,\ y = 0,\ r = 1$ Apply the definition of the secant function.

$$\sec\pi = \frac{r}{x} = \frac{1}{-1} = -1$$

12. $\theta = \pi$ radians

The terminal side of the angle is on the negative *x*-axis. Select the point $P = (-1, 0)$: $x = -1, y = 0,\ r = 1$

Apply the definition of the cosecant function.

$$\csc\pi = \frac{r}{y} = \frac{1}{0}, \text{ undefined}$$

13. $\theta = \frac{3\pi}{2}$ radians

The terminal side of the angle is on the negative *y*-axis. Select the point $P = (0, -1)$: $x = 0,\ y = -1,\ r = 1$ Apply the definition of the tangent function. $\tan\frac{3\pi}{2} = \frac{y}{x} = \frac{-1}{0}$, undefined

14. $\theta = \frac{3\pi}{2}$ radians

The terminal side of the angle is on the negative *y*-axis. Select the point $P = (0, -1)$: $x = 0, y = -1,\ r = 1$

Apply the definition of the cosine function.

$$\cos\frac{3\pi}{2} = \frac{x}{r} = \frac{0}{1} = 0$$

15. $\theta = \frac{\pi}{2}$ radians

The terminal side of the angle is on the positive *y*-axis. Select the point $P = (0, 1)$: $x = 0,\ y = 1,\ r = 1$ Apply the definition of the cotangent function. $\cot\frac{\pi}{2} = \frac{x}{y} = \frac{0}{1} = 0$

16. $\theta = \frac{\pi}{2}$ radians

The terminal side of the angle is on the positive *y*-axis. Select the point $P = (0, 1)$: $x = 0, y = 1,\ r = 1$

Apply the definition of the tangent function.

$$\tan\frac{\pi}{2} = \frac{y}{x} = \frac{1}{0}, \text{ undefined}$$

17. Because $\sin\theta > 0$, θ cannot lie in quadrant III or quadrant IV; the sine function is negative in those quadrants. Thus, with $\sin\theta > 0$, θ lies in quadrant I or quadrant II. We are also given that $\cos\theta > 0$. Because quadrant I is the only quadrant in which the cosine is positive and sine is positive, we conclude that θ lies in quadrant I.

18. Because $\sin\theta < 0$, θ cannot lie in quadrant I or quadrant II; the sine function is positive in those two quadrants. Thus, with $\sin\theta < 0$, θ lies in quadrant III or quadrant IV. We are also given that $\cos\theta > 0$. Because quadrant IV is the only quadrant in which the cosine is positive and the sine is negative, we conclude that θ lies in quadrant IV.

19. Because $\sin\theta < 0$, θ cannot lie in quadrant I or quadrant II; the sine function is positive in those two quadrants. Thus, with $\sin\theta < 0$, θ lies in quadrant III or quadrant IV. We are also given that $\cos\theta < 0$. Because quadrant III is the only quadrant in which the cosine is positive and the sine is negative, we conclude that θ lies in quadrant III.

20. Because $\tan\theta < 0$, θ cannot lie in quadrant I or quadrant III; the tangent function is positive in those two quadrants. Thus, with $\tan\theta < 0$, θ lies in quadrant II or quadrant IV. We are also given that $\sin\theta < 0$. Because quadrant IV is the only quadrant in which the sine is negative and the tangent is negative, we conclude that θ lies in quadrant IV.

21. Because $\tan\theta < 0$, θ cannot lie in quadrant I or quadrant III; the tangent function is positive in those quadrants. Thus, with $\tan\theta < 0$, θ lies in quadrant II or quadrant IV. We are also given that $\cos\theta < 0$. Because quadrant II is the only quadrant in which the cosine is negative and the tangent is negative, we conclude that θ lies in quadrant II.

22. Because $\cot\theta > 0$, θ cannot lie in quadrant II or quadrant IV; the cotangent function is negative in those two quadrants. Thus, with $\cot\theta > 0$, θ lies in quadrant I or quadrant III. We are also given that $\sec\theta < 0$. Because quadrant III is the only quadrant in which the secant is negative and the cotangent is positive, we conclude that θ lies in quadrant III.

23. In quadrant III x is negative and y is negative. Thus, $\cos\theta = -\frac{3}{5} = \frac{x}{r} = \frac{-3}{5}$, $x = -3$, $r = 5$. Furthermore,

$$r^2 = x^2 + y^2$$
$$5^2 = (-3)^2 + y^2$$
$$y^2 = 25 - 9 = 16$$
$$y = -\sqrt{16} = -4$$

Now that we know x, y, and r, we can find the remaining trigonometric functions of θ.

$$\sin\theta = \frac{y}{r} = \frac{-4}{5} = -\frac{4}{5}$$
$$\tan\theta = \frac{y}{x} = \frac{-4}{-3} = \frac{4}{3}$$
$$\csc\theta = \frac{r}{y} = \frac{5}{-4} = -\frac{5}{4}$$
$$\sec\theta = \frac{r}{x} = \frac{5}{-3} = -\frac{5}{3}$$
$$\cot\theta = \frac{x}{y} = \frac{-3}{-4} = \frac{3}{4}$$

24. In quadrant III, x is negative and y is negative. Thus, $\sin\theta = -\frac{12}{13} = \frac{y}{r} = \frac{-12}{13}$, $y = -12$, $r = 13$.

Furthermore,

$$x^2 + y^2 = r^2$$
$$x^2 + (-12)^2 = 13^2$$
$$x^2 = 169 - 144 = 25$$
$$x = -\sqrt{25} = -5$$

Now that we know x, y, and r, we can find the remaining trigonometric functions of θ.

$$\cos\theta = \frac{x}{r} = \frac{-5}{13} = -\frac{5}{13}$$
$$\tan\theta = \frac{y}{x} = \frac{-12}{-5} = \frac{12}{5}$$
$$\csc\theta = \frac{r}{y} = \frac{13}{-12} = -\frac{13}{12}$$
$$\sec\theta = \frac{r}{x} = \frac{13}{-5} = -\frac{13}{5}$$
$$\cot\theta = \frac{x}{y} = \frac{-5}{-12} = \frac{5}{12}$$

25. In quadrant II x is negative and y is positive. Thus,
$\sin\theta = \frac{5}{13} = \frac{y}{r}$, $y = 5$, $r = 13$. Furthermore,

$$x^2 + y^2 = r^2$$
$$x^2 + 5^2 = 13^2$$
$$x^2 = 169 - 25 = 144$$
$$x = -\sqrt{144} = -12$$

Now that we know x, y, and r, we can find the remaining trigonometric functions of θ.

$$\cos\theta = \frac{x}{r} = \frac{-12}{13} = -\frac{12}{13}$$
$$\tan\theta = \frac{y}{x} = \frac{5}{-12} = -\frac{5}{12}$$
$$\csc\theta = \frac{r}{y} = \frac{13}{5}$$
$$\sec\theta = \frac{r}{x} = \frac{13}{-12} = -\frac{13}{12}$$
$$\cot\theta = \frac{x}{y} = \frac{-12}{5} = -\frac{12}{5}$$

26. In quadrant IV, x is positive and y is negative. Thus,
$\cos\theta = \frac{4}{5} = \frac{x}{r}$, $x = 4$, $r = 5$. Furthermore,

$$x^2 + y^2 = r^2$$
$$4^2 + y^2 = 5^2$$
$$y^2 = 25 - 16 = 9$$
$$y = -\sqrt{9} = -3$$

Now that we know x, y, and r, we can find the remaining trigonometric functions of θ.

$$\sin\theta = \frac{y}{r} = \frac{-3}{5} = -\frac{3}{5}$$
$$\tan\theta = \frac{y}{x} = \frac{-3}{4} = -\frac{3}{4}$$
$$\csc\theta = \frac{r}{y} = \frac{5}{-3} = -\frac{5}{3}$$
$$\sec\theta = \frac{r}{x} = \frac{5}{4}$$
$$\cot\theta = \frac{x}{y} = \frac{4}{-3} = -\frac{4}{3}$$

27. Because $270° < \theta < 360°$, θ is in quadrant IV. In quadrant IV x is positive and y is negative. Thus,
$\cos\theta = \frac{8}{17} = \frac{x}{r}$, $x = 8$,
$r = 17$. Furthermore

$$x^2 + y^2 = r^2$$
$$8^2 + y^2 = 17^2$$
$$y^2 = 289 - 64 = 225$$
$$y = -\sqrt{225} = -15$$

Now that we know x, y, and r, we can find the remaining trigonometric functions of θ.

$$\sin\theta = \frac{y}{r} = \frac{-15}{17} = -\frac{15}{17}$$
$$\tan\theta = \frac{y}{x} = \frac{-15}{8} = -\frac{15}{8}$$
$$\csc\theta = \frac{r}{y} = \frac{17}{-15} = -\frac{17}{15}$$
$$\sec\theta = \frac{r}{x} = \frac{17}{8}$$
$$\cot\theta = \frac{x}{y} = \frac{8}{-15} = -\frac{8}{15}$$

28. Because $270° < \theta < 360°$, θ is in quadrant IV. In quadrant IV, x is positive and y is negative. Thus,
$\cos\theta = \frac{1}{3} = \frac{x}{r}$, $x = 1$, $r = 3$. Furthermore,

$$x^2 + y^2 = r^2$$
$$1^2 + y^2 = 3^2$$
$$y^2 = 9 - 1 = 8$$
$$y = -\sqrt{8} = -2\sqrt{2}$$

Now that we know x, y, and r, we can find the remaining trigonometric functions of θ.

$$\sin\theta = \frac{y}{r} = \frac{-2\sqrt{2}}{3} = -\frac{2\sqrt{2}}{3}$$
$$\tan\theta = \frac{y}{x} = \frac{-2\sqrt{2}}{1} = -2\sqrt{2}$$
$$\csc\theta = \frac{r}{y} = \frac{3}{-2\sqrt{2}} = \frac{3}{-2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = -\frac{3\sqrt{2}}{4}$$
$$\sec\theta = \frac{r}{x} = \frac{3}{1} = 3$$
$$\cot\theta = \frac{x}{y} = \frac{1}{-2\sqrt{2}} = \frac{1}{-2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = -\frac{\sqrt{2}}{4}$$

29. Because the tangent is negative and the sine is positive, θ lies in quadrant II. In quadrant II, x is negative and y is positive. Thus,

$\tan\theta = -\frac{2}{3} = \frac{y}{x} = \frac{2}{-3}$, $x = -3$, $y = 2$. Furthermore,

$r = \sqrt{x^2 + y^2} = \sqrt{(-3)^2 + 2^2} = \sqrt{9+4} = \sqrt{13}$

Now that we know x, y, and r, we can find the remaining trigonometric functions of θ.

$$\sin\theta = \frac{y}{r} = \frac{2}{\sqrt{13}} = \frac{2}{\sqrt{13}} \cdot \frac{\sqrt{13}}{\sqrt{13}} = \frac{2\sqrt{13}}{13}$$

$$\cos\theta = \frac{x}{r} = \frac{-3}{\sqrt{13}} = \frac{-3}{\sqrt{13}} \cdot \frac{\sqrt{13}}{\sqrt{13}} = -\frac{3\sqrt{13}}{13}$$

$$\csc\theta = \frac{r}{y} = \frac{\sqrt{13}}{2}$$

$$\sec\theta = \frac{r}{x} = \frac{\sqrt{13}}{-3} = -\frac{\sqrt{13}}{3}$$

$$\cot\theta = \frac{x}{y} = \frac{-3}{2} = -\frac{3}{2}$$

30. Because the tangent is negative and the sine is positive, θ lies in quadrant II. In quadrant II, x is negative and y is positive. Thus,

$\tan\theta = -\frac{1}{3} = \frac{y}{x} = \frac{1}{-3}$, $y = 1$, $x = -3$. Furthermore,

$r = \sqrt{x^2 + y^2} = \sqrt{(-3)^2 + 1^2} = \sqrt{9+1} = \sqrt{10}$

Now that we know x, y, and r, we can find the remaining trigonometric functions of θ.

$$\sin\theta = \frac{y}{r} = \frac{1}{\sqrt{10}} = \frac{1}{\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}} = \frac{\sqrt{10}}{10}$$

$$\cos\theta = \frac{x}{r} = \frac{-3}{\sqrt{10}} = \frac{-3}{\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}} = -\frac{3\sqrt{10}}{10}$$

$$\csc\theta = \frac{r}{y} = \frac{\sqrt{10}}{1} = \sqrt{10}$$

$$\sec\theta = \frac{r}{x} = \frac{\sqrt{10}}{-3} = -\frac{\sqrt{10}}{3}$$

$$\cot\theta = \frac{x}{y} = \frac{-3}{1} = -3$$

31. Because the tangent is positive and the cosine is negative, θ lies in quadrant III. In quadrant III, x is negative and y is negative. Thus, $\tan\theta = \frac{4}{3} = \frac{y}{x} = \frac{-4}{-3}$, $x = -3$, $y = -4$. Furthermore,

$r = \sqrt{x^2 + y^2} = \sqrt{(-3)^2 + (-4)^2} = \sqrt{9+16}$
$= \sqrt{25} = 5$

Now that we know x, y, and r, we can find the remaining trigonometric functions of θ.

$$\sin\theta = \frac{y}{r} = \frac{-4}{5} = -\frac{4}{5}$$

$$\cos\theta = \frac{x}{r} = \frac{-3}{5} = -\frac{3}{5}$$

$$\csc\theta = \frac{r}{y} = \frac{5}{-4} = -\frac{5}{4}$$

$$\sec\theta = \frac{r}{x} = \frac{5}{-3} = -\frac{5}{3}$$

$$\cot\theta = \frac{x}{y} = \frac{-3}{-4} = \frac{3}{4}$$

32. Because the tangent is positive and the cosine is negative, θ lies in quadrant III. In quadrant III, x is negative and y is negative. Thus,

$\tan\theta = \frac{5}{12} = \frac{y}{x} = \frac{-5}{-12}$, $x = -12$, $y = -5$. Furthermore,

$r = \sqrt{x^2 + y^2} = \sqrt{(-12)^2 + (-5)^2} = \sqrt{144+25}$
$= \sqrt{169} = 13$

Now that we know x, y, and r, we can find the remaining trigonometric functions of θ.

$$\sin\theta = \frac{y}{r} = \frac{-5}{13} = -\frac{5}{13}$$

$$\cos\theta = \frac{x}{r} = \frac{-12}{13} = -\frac{12}{13}$$

$$\csc\theta = \frac{r}{y} = \frac{13}{-5} = -\frac{13}{5}$$

$$\sec\theta = \frac{r}{x} = \frac{13}{-12} = -\frac{13}{12}$$

$$\cot\theta = \frac{x}{y} = \frac{-12}{-5} = \frac{12}{5}$$

33. Because the secant is negative and the tangent is positive, θ lies in quadrant III. In quadrant III, x is negative and y is negative. Thus,

$\sec\theta = -3 = \frac{r}{x} = \frac{3}{-1}$, $x = -1$, $r = 3$. Furthermore,

$$\begin{aligned} x^2 + y^2 &= r^2 \\ (-1)^2 + y^2 &= 3^2 \\ y^2 &= 9 - 1 = 8 \\ y &= -\sqrt{8} = -2\sqrt{2} \end{aligned}$$

Now that we know x, y, and r, we can find the remaining trigonometric functions of θ.

$$\sin\theta = \frac{y}{r} = \frac{-2\sqrt{2}}{3} = -\frac{2\sqrt{2}}{3}$$

$$\cos\theta = \frac{x}{r} = \frac{-1}{3} = -\frac{1}{3}$$

$$\tan\theta = \frac{y}{x} = \frac{-2\sqrt{2}}{-1} = 2\sqrt{2}$$

$$\csc\theta = \frac{r}{y} = \frac{3}{-2\sqrt{2}} = \frac{3}{-2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = -\frac{3\sqrt{2}}{4}$$

$$\cot\theta = \frac{x}{y} = \frac{-1}{-2\sqrt{2}} = \frac{1}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{4}$$

34. Because the cosecant is negative and the tangent is positive, θ lies in quadrant III. In quadrant III, x is negative and y is negative. Thus,

$\csc\theta = -4 = \frac{r}{y} = \frac{4}{-1}$, $y = -1$, $r = 4$. Furthermore,

$$\begin{aligned} x^2 + y^2 &= r^2 \\ x^2 + (-1)^2 &= 4^2 \\ x^2 &= 16 - 1 = 15 \\ x &= -\sqrt{15} \end{aligned}$$

Now that we know x, y, and r, we can find the remaining trigonometric functions of θ.

$$\sin\theta = \frac{y}{r} = \frac{-1}{4} = -\frac{1}{4}$$

$$\cos\theta = \frac{x}{r} = \frac{-\sqrt{15}}{4} = -\frac{\sqrt{15}}{4}$$

$$\tan\theta = \frac{y}{x} = \frac{-1}{-\sqrt{15}} = \frac{1}{\sqrt{15}} \cdot \frac{\sqrt{15}}{\sqrt{15}} = \frac{\sqrt{15}}{15}$$

$$\sec\theta = \frac{r}{x} = \frac{4}{-\sqrt{15}} = -\frac{4}{\sqrt{15}} \cdot \frac{\sqrt{15}}{\sqrt{15}} = -\frac{4\sqrt{15}}{15}$$

$$\cot\theta = \frac{x}{y} = \frac{-\sqrt{15}}{-1} = \sqrt{15}$$

35. Because 160° lies between 90° and 180°, it is in quadrant II. The reference angle is $\theta' = 180° - 160° = 20°$.

36. Because 170° lies between 90° and 180°, it is in quadrant II. The reference angle is $\theta' = 180° - 170° = 10°$.

37. Because 205° lies between 180° and 270°, it is in quadrant III. The reference angle is $\theta' = 205° - 180° = 25°$.

38. Because 210° lies between 180° and 270°, it is in quadrant III. The reference angle is $\theta' = 210° - 180° = 30°$.

39. Because 355° lies between 270° and 360°, it is in quadrant IV. The reference angle is $\theta' = 360° - 355° = 5°$.

40. Because 351° lies between 270° and 360°, it is in quadrant IV. The reference angle is $\theta' = 360° - 351° = 9°$.

41. Because $\frac{7\pi}{4}$ lies between $\frac{3\pi}{2} = \frac{6\pi}{4}$ and $2\pi = \frac{8\pi}{4}$, it is in quadrant IV. The reference angle is $\theta' = 2\pi - \frac{7\pi}{4} = \frac{8\pi}{4} - \frac{7\pi}{4} = \frac{\pi}{4}$.

42. Because $\frac{5\pi}{4}$ lies between $\pi = \frac{4\pi}{4}$ and $\frac{3\pi}{2} = \frac{6\pi}{4}$, it is in quadrant III. The reference angle is $\theta' = \frac{5\pi}{4} - \pi = \frac{5\pi}{4} - \frac{4\pi}{4} = \frac{\pi}{4}$.

43. Because $\frac{5\pi}{6}$ lies between $\frac{\pi}{2} = \frac{3\pi}{6}$ and $\pi = \frac{6\pi}{6}$, it is in quadrant II. The reference angle is $\theta' = \pi - \frac{5\pi}{6} = \frac{6\pi}{6} - \frac{5\pi}{6} = \frac{\pi}{6}$.

44. Because $\frac{5\pi}{7} = \frac{10\pi}{14}$ lies between $\frac{\pi}{2} = \frac{7\pi}{14}$ and $\pi = \frac{14\pi}{14}$, it is in quadrant II. The reference angle is $\theta' = \pi - \frac{5\pi}{7} = \frac{7\pi}{7} - \frac{5\pi}{7} = \frac{2\pi}{7}$.

45. $-150° + 360° = 210°$
Because the angle is in quadrant III, the reference angle is $\theta' = 210° - 180° = 30°$.

46. $-250° + 360° = 110°$
Because the angle is in quadrant II, the reference angle is $\theta' = 180° - 110° = 70°$.

47. $-335° + 360° = 25°$
Because the angle is in quadrant I, the reference angle is $\theta' = 25°$.

48. $-359° + 360° = 1°$
Because the angle is in quadrant I, the reference angle is $\theta' = 1°$.

49. Because 4.7 lies between $\pi \approx 3.14$ and $\frac{3\pi}{2} \approx 4.71$, it is in quadrant III. The reference angle is $\theta' = 4.7 - \pi \approx 1.56$.

50. Because 5.5 lies between $\frac{3\pi}{2} \approx 4.71$ and $2\pi \approx 6.28$, it is in quadrant IV. The reference angle is $\theta' = 2\pi - 5.5 \approx 0.78$.

51. $565° - 360° = 205°$
Because the angle is in quadrant III, the reference angle is $\theta' = 205° - 180° = 25°$.

52. $553° - 360° = 193°$
Because the angle is in quadrant III, the reference angle is $\theta' = 193° - 180° = 13°$.

53. $\frac{17\pi}{6} - 2\pi = \frac{17\pi}{6} - \frac{12\pi}{6} = \frac{5\pi}{6}$
Because the angle is in quadrant II, the reference angle is $\theta' = \pi - \frac{5\pi}{6} = \frac{\pi}{6}$.

54. $\frac{11\pi}{4} - 2\pi = \frac{11\pi}{4} - \frac{8\pi}{4} = \frac{3\pi}{4}$
Because the angle is in quadrant II, the reference angle is $\theta' = \pi - \frac{3\pi}{4} = \frac{\pi}{4}$.

55. $\frac{23\pi}{4} - 4\pi = \frac{23\pi}{4} - \frac{16\pi}{4} = \frac{7\pi}{4}$
Because the angle is in quadrant IV, the reference angle is $\theta' = 2\pi - \frac{7\pi}{4} = \frac{\pi}{4}$.

56. $\frac{17\pi}{3} - 4\pi = \frac{17\pi}{3} - \frac{12\pi}{3} = \frac{5\pi}{3}$
Because the angle is in quadrant IV, the reference angle is $\theta' = 2\pi - \frac{5\pi}{3} = \frac{\pi}{3}$.

57. $-\frac{11\pi}{4} + 4\pi = -\frac{11\pi}{4} + \frac{16\pi}{4} = \frac{5\pi}{4}$
Because the angle is in quadrant III, the reference angle is $\theta' = \frac{5\pi}{4} - \pi = \frac{\pi}{4}$.

58. $-\frac{17\pi}{6} + 4\pi = -\frac{17\pi}{6} + \frac{24\pi}{6} = \frac{7\pi}{6}$
Because the angle is in quadrant III, the reference angle is $\theta' = \frac{7\pi}{6} - \pi = \frac{\pi}{6}$.

59. $-\frac{25\pi}{6} + 6\pi = -\frac{25\pi}{6} + \frac{36\pi}{6} = \frac{11\pi}{6}$
Because the angle is in quadrant IV, the reference angle is $\theta' = 2\pi - \frac{11\pi}{6} = \frac{\pi}{6}$.

60. $-\frac{13\pi}{3} + 6\pi = -\frac{13\pi}{3} + \frac{18\pi}{3} = \frac{5\pi}{3}$
Because the angle is in quadrant IV, the reference angle is $\theta' = 2\pi - \frac{5\pi}{3} = \frac{\pi}{3}$.

61. 225° lies in quadrant III. The reference angle is $\theta' = 225° - 180° = 45°$.
$\cos 45° = \frac{\sqrt{2}}{2}$
Because the cosine is negative in quadrant III, $\cos 225° = -\cos 45° = -\frac{\sqrt{2}}{2}$.

62. 300° lies in quadrant IV. The reference angle is $\theta' = 360° - 300° = 60°$.
$\sin 60° = \frac{\sqrt{3}}{2}$
Because the sine is negative in quadrant IV, $\sin 300° = -\sin 60° = -\frac{\sqrt{3}}{2}$.

63. 210° lies in quadrant III. The reference angle is $\theta' = 210° - 180° = 30°$.
$\tan 30° = \frac{\sqrt{3}}{3}$
Because the tangent is positive in quadrant III, $\tan 210° = \tan 30° = \frac{\sqrt{3}}{3}$.

64. 240° lies in quadrant III. The reference angle is $\theta' = 240° - 180° = 60°$.
$\sec 60° = 2$
Because the secant is negative in quadrant III, $\sec 240° = -\sec 60° - 2$.

65. 420° lies in quadrant I. The reference angle is $\theta' = 420° - 360° = 60°$.
$\tan 60° = \sqrt{3}$
Because the tangent is positive in quadrant I, $\tan 420° = \tan 60° = \sqrt{3}$.

66. 405° lies in quadrant I. The reference angle is $\theta' = 405° - 360° = 45°$.
$\tan 45° = 1$
Because the tangent is positive in quadrant I, $\tan 405° = \tan 45° = 1$.

67. $\frac{2\pi}{3}$ lies in quadrant II. The reference angle is

$\theta' = \pi - \frac{2\pi}{3} = \frac{3\pi}{3} - \frac{2\pi}{3} = \frac{\pi}{3}$.

$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

Because the sine is positive in quadrant II,

$\sin\frac{2\pi}{3} = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$.

68. $\frac{3\pi}{4}$ lies in quadrant II. The reference angle is

$\theta' = \pi - \frac{3\pi}{4} = \frac{4\pi}{4} - \frac{3\pi}{4} = \frac{\pi}{4}$.

$\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

Because the cosine is negative in quadrant II,

$\cos\frac{3\pi}{4} = -\cos\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$.

69. $\frac{7\pi}{6}$ lies in quadrant III. The reference angle is

$\theta' = \frac{7\pi}{6} - \pi = \frac{7\pi}{6} - \frac{6\pi}{6} = \frac{\pi}{6}$.

$\csc\frac{\pi}{6} = 2$

Because the cosecant is negative in quadrant III,

$\csc\frac{7\pi}{6} = -\csc\frac{\pi}{6} = -2$.

70. $\frac{7\pi}{4}$ lies in quadrant IV. The reference angle is

$\theta' = 2\pi - \frac{7\pi}{4} = \frac{8\pi}{4} - \frac{7\pi}{4} = \frac{\pi}{4}$.

$\cot\frac{\pi}{4} = 1$

Because the cotangent is negative in quadrant IV,

$\cot\frac{7\pi}{4} = -\cot\frac{\pi}{4} = -1$.

71. $\frac{9\pi}{4}$ lies in quadrant I. The reference angle is

$\theta' = \frac{9\pi}{4} - 2\pi = \frac{9\pi}{4} - \frac{8\pi}{4} = \frac{\pi}{4}$.

$\tan\frac{\pi}{4} = 1$

Because the tangent is positive in quadrant I,

$\tan\frac{9\pi}{4} = \tan\frac{\pi}{4} = 1$

72. $\frac{9\pi}{2}$ lies on the positive *y*-axis. The reference angle is

$\theta' = \frac{9\pi}{2} - 4\pi = \frac{9\pi}{2} - \frac{8\pi}{2} = \frac{\pi}{2}$.

Because $\tan\frac{\pi}{2}$ is undefined, $\tan\frac{9\pi}{2}$ is also undefined.

73. –240° lies in quadrant II. The reference angle is $\theta' = 240° - 180° = 60°$.

$\sin 60° = \frac{\sqrt{3}}{2}$

Because the sine is positive in quadrant II,

$\sin(-240°) = \sin 60° = \frac{\sqrt{3}}{2}$.

74. –225° lies in quadrant II. The reference angle is $\theta' = 225° - 180° = 45°$.

$\sin 45° = \frac{\sqrt{2}}{2}$

Because the sine is positive in quadrant II,

$\sin(-225°) = \sin 45° = \frac{\sqrt{2}}{2}$.

75. $-\frac{\pi}{4}$ lies in quadrant IV. The reference angle is

$\theta' = \frac{\pi}{4}$.

$\tan\frac{\pi}{4} = 1$

Because the tangent is negative in quadrant IV,

$\tan\left(-\frac{\pi}{4}\right) = -\tan\frac{\pi}{4} = -1$

76. $-\frac{\pi}{6}$ lies in quadrant IV. The reference angle is

$\theta = \frac{\pi}{6}$. $\quad \tan\frac{\pi}{6} = \frac{\sqrt{3}}{3}$

Because the tangent is negative in quadrant IV,

$\tan\left(-\frac{\pi}{6}\right) = -\frac{\sqrt{3}}{3}$.

77. $\sec 495° = \sec 135° = -\sqrt{2}$

78. $\sec 510° = \sec 150° = -\frac{2\sqrt{3}}{3}$

79. $\cot\frac{19\pi}{6} = \cot\frac{7\pi}{6} = \sqrt{3}$

80. $\cot\frac{13\pi}{3} = \cot\frac{\pi}{3} = \frac{\sqrt{3}}{3}$

81. $\cos\frac{23\pi}{4} = \cos\frac{7\pi}{4} = \frac{\sqrt{2}}{2}$

82. $\cos\frac{35\pi}{6} = \cos\frac{11\pi}{6} = \frac{\sqrt{3}}{2}$

83. $\tan\left(-\frac{17\pi}{6}\right) = \tan\frac{7\pi}{6} = \frac{\sqrt{3}}{3}$

84. $\tan\left(-\frac{11\pi}{4}\right) = \tan\frac{\pi}{4} = 1$

85. $\sin\left(-\frac{17\pi}{3}\right) = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

86. $\sin\left(-\frac{35\pi}{6}\right) = \sin\frac{\pi}{6} = \frac{1}{2}$

87. $\sin\frac{\pi}{3}\cos\pi - \cos\frac{\pi}{3}\sin\frac{3\pi}{2}$

$= \left(\frac{\sqrt{3}}{2}\right)(-1) - \left(\frac{1}{2}\right)(-1)$

$= -\frac{\sqrt{3}}{2} + \frac{1}{2}$

$= \frac{1-\sqrt{3}}{2}$

88. $\sin\frac{\pi}{4}\cos 0 - \sin\frac{\pi}{6}\cos\pi$

$= \left(\frac{\sqrt{2}}{2}\right)(1) - \left(\frac{1}{2}\right)(-1)$

$= \frac{\sqrt{2}}{2} + \frac{1}{2}$

$= \frac{\sqrt{2}+1}{2}$

89. $\sin\frac{11\pi}{4}\cos\frac{5\pi}{6} + \cos\frac{11\pi}{4}\sin\frac{5\pi}{6}$

$= \left(\frac{\sqrt{2}}{2}\right)\left(-\frac{\sqrt{3}}{2}\right) + \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$

$= -\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$

$= -\frac{\sqrt{6}+\sqrt{2}}{4}$

90. $\sin\frac{17\pi}{3}\cos\frac{5\pi}{4} + \cos\frac{17\pi}{3}\sin\frac{5\pi}{4}$

$= \left(-\frac{\sqrt{3}}{2}\right)\left(-\frac{\sqrt{2}}{2}\right) + \left(\frac{1}{2}\right)\left(-\frac{\sqrt{2}}{2}\right)$

$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$

$= \frac{\sqrt{6}-\sqrt{2}}{4}$

91. $\sin\frac{3\pi}{2}\tan\left(-\frac{15\pi}{4}\right)-\cos\left(-\frac{5\pi}{3}\right)$

$=(-1)(1)-\left(\frac{1}{2}\right)$

$=-1-\frac{1}{2}$

$=-\frac{2}{2}-\frac{1}{2}$

$=-\frac{3}{2}$

92. $\sin\frac{3\pi}{2}\tan\left(-\frac{8\pi}{3}\right)+\cos\left(-\frac{5\pi}{6}\right)$

$=(-1)\left(\sqrt{3}\right)+\left(-\frac{\sqrt{3}}{2}\right)$

$=-\sqrt{3}-\frac{\sqrt{3}}{2}$

$=-\frac{2\sqrt{3}}{2}-\frac{\sqrt{3}}{2}$

$=-\frac{3\sqrt{3}}{2}$

93. $f\left(\frac{4\pi}{3}+\frac{\pi}{6}\right)+f\left(\frac{4\pi}{3}\right)+f\left(\frac{\pi}{6}\right)$

$=\sin\left(\frac{4\pi}{3}+\frac{\pi}{6}\right)+\sin\frac{4\pi}{3}+\sin\frac{\pi}{6}$

$=\sin\frac{3\pi}{2}+\sin\frac{4\pi}{3}+\sin\frac{\pi}{6}$

$=(-1)+\left(-\frac{\sqrt{3}}{2}\right)+\left(\frac{1}{2}\right)$

$=-\frac{\sqrt{3}+1}{2}$

94. $g\left(\frac{5\pi}{6}+\frac{\pi}{6}\right)+g\left(\frac{5\pi}{6}\right)+g\left(\frac{\pi}{6}\right)$

$=\cos\left(\frac{5\pi}{6}+\frac{\pi}{6}\right)+\cos\frac{5\pi}{6}+\cos\frac{\pi}{6}$

$=\cos\pi+\cos\frac{5\pi}{6}+\cos\frac{\pi}{6}$

$=(-1)+\left(-\frac{\sqrt{3}}{2}\right)+\left(\frac{\sqrt{3}}{2}\right)$

$=-1$

95. $(h\circ g)\left(\frac{17\pi}{3}\right)=h\left(g\left(\frac{17\pi}{3}\right)\right)$

$=2\left(\cos\left(\frac{17\pi}{3}\right)\right)$

$=2\left(\frac{1}{2}\right)$

$=1$

96. $(h\circ f)\left(\frac{11\pi}{4}\right)=h\left(f\left(\frac{11\pi}{4}\right)\right)$

$=2\left(\sin\left(\frac{11\pi}{4}\right)\right)$

$=2\left(\frac{\sqrt{2}}{2}\right)$

$=\sqrt{2}$

97. The average rate of change is the slope of the line through the points $(x_1, f(x_1))$ and $(x_2, f(x_2))$

$$m=\frac{f(x_2)-f(x_1)}{x_2-x_1}$$

$$=\frac{\sin\left(\frac{3\pi}{2}\right)-\sin\left(\frac{5\pi}{4}\right)}{\frac{3\pi}{2}-\frac{5\pi}{4}}$$

$$=\frac{-1-\left(-\frac{\sqrt{2}}{2}\right)}{\frac{\pi}{4}}$$

$$=\frac{-1+\frac{\sqrt{2}}{2}}{\frac{\pi}{4}}$$

$$=\frac{4\left(-1+\frac{\sqrt{2}}{2}\right)}{4\left(\frac{\pi}{4}\right)}$$

$$=\frac{2\sqrt{2}-4}{\pi}$$

98. The average rate of change is the slope of the line through the points $\left(x_1, g(x_1)\right)$ and $\left(x_2, g(x_2)\right)$

$$\begin{aligned} m &= \frac{g(x_2)-g(x_1)}{x_2-x_1} \\ &= \frac{\cos(\pi)-\cos\left(\frac{3\pi}{4}\right)}{\pi-\frac{3\pi}{4}} \\ &= \frac{-1-\left(-\frac{\sqrt{2}}{2}\right)}{\frac{\pi}{4}} \\ &= \frac{4\left(-1+\frac{\sqrt{2}}{2}\right)}{4\left(\frac{\pi}{4}\right)} \\ &= \frac{2\sqrt{2}-4}{\pi} \end{aligned}$$

99. $\sin\theta = \frac{\sqrt{2}}{2}$ when the reference angle is $\frac{\pi}{4}$ and θ is in quadrants I or II.

QI	QII
$\theta = \frac{\pi}{4}$	$\theta = \pi - \frac{\pi}{4}$
	$= \frac{3\pi}{4}$

$\theta = \frac{\pi}{4}, \frac{3\pi}{4}$

100. $\cos\theta = \frac{1}{2}$ when the reference angle is $\frac{\pi}{3}$ and θ is in quadrants I or IV.

QI	QIV
$\theta = \frac{\pi}{3}$	$\theta = 2\pi - \frac{\pi}{3}$
	$= \frac{5\pi}{3}$

$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$

101. $\sin\theta = -\frac{\sqrt{2}}{2}$ when the reference angle is $\frac{\pi}{4}$ and θ is in quadrants III or IV.

QIII	QIV
$\theta = \pi + \frac{\pi}{4}$	$\theta = 2\pi - \frac{\pi}{4}$
$= \frac{5\pi}{4}$	$= \frac{7\pi}{4}$

$\theta = \frac{5\pi}{4}, \frac{7\pi}{4}$

102. $\cos\theta = -\frac{1}{2}$ when the reference angle is $\frac{\pi}{3}$ and θ is in quadrants II or III.

QII	QIII
$\theta = \pi - \frac{\pi}{3}$	$\theta = \pi + \frac{\pi}{3}$
$= \frac{2\pi}{3}$	$= \frac{4\pi}{3}$

$\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$

103. $\tan\theta = -\sqrt{3}$ when the reference angle is $\frac{\pi}{3}$ and θ is in quadrants II or IV.

QII	QIV
$\theta = \pi - \frac{\pi}{3}$	$\theta = 2\pi - \frac{\pi}{3}$
$= \frac{2\pi}{3}$	$= \frac{5\pi}{3}$

$\theta = \frac{2\pi}{3}, \frac{5\pi}{3}$

104. $\tan\theta = -\frac{\sqrt{3}}{3}$ when the reference angle is $\frac{\pi}{6}$ and θ is in quadrants II or IV.

QII	QIV
$\theta = \pi - \frac{\pi}{6}$	$\theta = 2\pi - \frac{\pi}{6}$
$= \frac{5\pi}{6}$	$= \frac{11\pi}{6}$

$\theta = \frac{5\pi}{6}, \frac{11\pi}{6}$

105. – 109. Answers may vary.

110. does not make sense; Explanations will vary. Sample explanation: Sine is defined for all values of the angle.

111. does not make sense; Explanations will vary. Sample explanation: Sine and cosecant have the same sign within any quadrant because they are reciprocals of each other.

112. does not make sense; Explanations will vary. Sample explanation: It is also possible that $y = -3$ and $x = -5.$

113. makes sense

114. $y = \frac{1}{2}\cos(4x + \pi)$

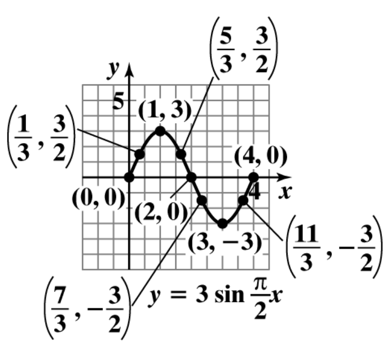
x	$-\frac{\pi}{4}$	$-\frac{\pi}{8}$	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$
y	$\frac{1}{2}$	0	$-\frac{1}{2}$	0	$\frac{1}{2}$

115. $y = 4\sin\left(2x - \frac{2\pi}{3}\right)$

x	$\frac{\pi}{3}$	$\frac{7\pi}{12}$	$\frac{5\pi}{6}$	$\frac{13\pi}{12}$	$\frac{4\pi}{3}$
y	0	4	0	–4	0

116. $y = 3\sin\frac{\pi}{2}x$

x	0	$\frac{1}{3}$	1	$\frac{5}{3}$	2	$\frac{7}{3}$	3	$\frac{11}{3}$	4
y	0	$\frac{3}{2}$	3	$\frac{3}{2}$	0	$-\frac{3}{2}$	–3	$-\frac{3}{2}$	0



Mid-Chapter 4 Check Point

1. $10° = 10° \cdot \frac{\pi \text{ radians}}{180°} = \frac{10\pi}{180}$ radians

$= \frac{\pi}{18}$ radians

2. $-105° = -105° \cdot \frac{\pi \text{ radians}}{180°} = -\frac{105\pi}{180}$ radians

$= -\frac{7\pi}{12}$ radians

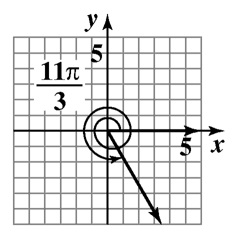
3. $\frac{5\pi}{12}$ radians $= \frac{5\pi \text{ radians}}{12} \cdot \frac{180^{\circ}}{\pi \text{ radians}} = 75^{\circ}$

4. $-\frac{13\pi}{20}$ radians $= -\frac{13\pi \text{ radians}}{20} \cdot \frac{180^{\circ}}{\pi \text{ radians}}$

$= -117^{\circ}$

5. a. $\frac{11\pi}{3} - 2\pi = \frac{11\pi}{3} - \frac{6\pi}{3} = \frac{5\pi}{3}$

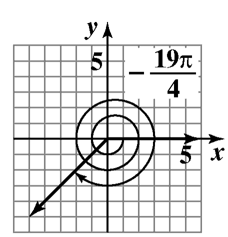
b.



c. Since $\frac{5\pi}{3}$ is in quadrant IV, the reference angle is $2\pi - \frac{5\pi}{3} = \frac{6\pi}{3} - \frac{5\pi}{3} = \frac{\pi}{3}$

6. a. $-\frac{19\pi}{4} + 6\pi = -\frac{19\pi}{4} + \frac{24\pi}{4} = \frac{5\pi}{4}$

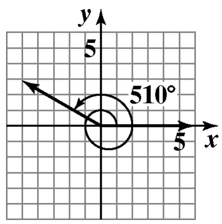
b.



c. Since $\frac{5\pi}{4}$ is in quadrant III, the reference angle is $\frac{5\pi}{4} - \pi = \frac{5\pi}{4} - \frac{4\pi}{4} = \frac{\pi}{4}$

7. **a.** $510° - 360° = 150°$

b.



c. Since $150°$ is in quadrant II, the reference angle is $180° - 150° = 30°$

8. $r = \sqrt{x^2 + y^2}$

$$r = \sqrt{\left(-\frac{3}{5}\right)^2 + \left(-\frac{4}{5}\right)^2} = \sqrt{\frac{9}{25} + \frac{16}{25}} = \sqrt{\frac{25}{25}} = 1$$

Now that we know x, y, and r, we can find the six trigonometric functions of θ.

$$\sin\theta = \frac{y}{r} = \frac{-\frac{4}{5}}{1} = -\frac{4}{5}$$

$$\cos\theta = \frac{x}{r} = \frac{-\frac{3}{5}}{1} = -\frac{3}{5}$$

$$\tan\theta = \frac{y}{x} = \frac{-\frac{4}{5}}{-\frac{3}{5}} = \frac{4}{3}$$

$$\csc\theta = \frac{r}{y} = \frac{1}{-\frac{4}{5}} = -\frac{5}{4}$$

$$\sec\theta = \frac{r}{x} = \frac{1}{-\frac{3}{5}} = -\frac{5}{3}$$

$$\cot\theta = \frac{x}{y} = \frac{-\frac{3}{5}}{-\frac{4}{5}} = \frac{3}{4}$$

9. Use the Pythagorean theorem to find b.

$$a^2 + b^2 = c^2$$
$$5^2 + b^2 = 6^2$$
$$25 + b^2 = 36$$
$$b^2 = 11$$
$$b = \sqrt{11}$$

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{5}{6}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{\sqrt{11}}{6}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{5\sqrt{11}}{11}$$

$$\csc\theta = \frac{\text{hypotenuse}}{\text{opposite}} = \frac{6}{5}$$

$$\sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}} = \frac{6}{\sqrt{11}} = \frac{6\sqrt{11}}{11}$$

$$\cot\theta = \frac{\text{adjacent}}{\text{opposite}} = \frac{\sqrt{11}}{5}$$

10. $r = \sqrt{x^2 + y^2}$

$$r = \sqrt{3^2 + (-2)^2} = \sqrt{9 + 4} = \sqrt{13}$$

Now that we know x, y, and r, we can find the six trigonometric functions of θ.

$$\sin\theta = \frac{y}{r} = \frac{-2}{\sqrt{13}} = -\frac{2\sqrt{13}}{13}$$

$$\cos\theta = \frac{x}{r} = \frac{3}{\sqrt{13}} = \frac{3\sqrt{13}}{13}$$

$$\tan\theta = \frac{y}{x} = \frac{-2}{3} = -\frac{2}{3}$$

$$\csc\theta = \frac{r}{y} = \frac{\sqrt{13}}{-2} = -\frac{\sqrt{13}}{2}$$

$$\sec\theta = \frac{r}{x} = \frac{\sqrt{13}}{3}$$

$$\cot\theta = \frac{x}{y} = \frac{3}{-2} = -\frac{3}{2}$$

11. Because the tangent is negative and the cosine is negative, θ is in quadrant II. In quadrant II, x is negative and y is positive. Thus,

$\tan\theta = -\frac{3}{4} = \frac{x}{y}, \quad x = -4,\ y = 3$. Furthermore,

$$r^2 = x^2 + y^2$$
$$r^2 = (-3)^2 + 4^2$$
$$r^2 = 9 + 16 = 25$$
$$r = 5$$

Now that we know x, y, and r, we can find the remaining trigonometric functions of θ.

$$\sin\theta = \frac{y}{r} = \frac{3}{5}$$
$$\cos\theta = \frac{x}{r} = \frac{-4}{5} = -\frac{4}{5}$$
$$\csc\theta = \frac{r}{y} = \frac{5}{3}$$
$$\sec\theta = \frac{r}{x} = \frac{5}{-3} = -\frac{5}{4}$$
$$\cot\theta = \frac{x}{y} = \frac{-3}{4} = -\frac{4}{3}$$

12. Since $\cos\theta = \frac{3}{7} = \frac{x}{r}, \quad x = 3,\ r = 7$. Furthermore,

$$x^2 + y^2 = r^2$$
$$3^2 + y^2 = 7^2$$
$$9 + y^2 = 49$$
$$y^2 = 40$$
$$y = \pm\sqrt{40} = \pm 2\sqrt{10}$$

Because the cosine is positive and the sine is negative, θ is in quadrant IV. In quadrant IV, x is positive and y is negative.

Therefore $y = -2\sqrt{10}$

Use x, y, and r to find the remaining trigonometric functions of θ.

$$\sin\theta = \frac{y}{r} = \frac{-2\sqrt{10}}{7} = -\frac{2\sqrt{10}}{7}$$
$$\tan\theta = \frac{y}{x} = \frac{-2\sqrt{10}}{3} = -\frac{2\sqrt{10}}{3}$$
$$\csc\theta = \frac{r}{y} = \frac{7}{-2\sqrt{10}} = -\frac{7\sqrt{10}}{20}$$
$$\sec\theta = \frac{r}{x} = \frac{7}{3}$$
$$\cot\theta = \frac{x}{y} = \frac{3}{-2\sqrt{10}} = -\frac{3\sqrt{10}}{20}$$

13. $$\tan\theta = \frac{\text{side opposite } \theta}{\text{side adjacent } \theta}$$
$$\tan 41° = \frac{a}{60}$$
$$a = 60\tan 41°$$
$$a \approx 52 \text{ cm}$$

14. $$\cos\theta = \frac{\text{side adjacent } \theta}{\text{hypotenuse}}$$
$$\cos 72° = \frac{250}{c}$$
$$c = \frac{250}{\cos 72°}$$
$$c \approx 809 \text{ m}$$

15. Since $\cos\theta = \frac{1}{6} = \frac{x}{r}, \quad x = 1,\ r = 6$. Furthermore,

$$x^2 + y^2 = r^2$$
$$1^2 + y^2 = 6^2$$
$$1 + y^2 = 36$$
$$y^2 = 35$$
$$y = \pm\sqrt{35}$$

Since θ is acute, $y = +\sqrt{35} = \sqrt{35}$

$$\cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta = \frac{y}{x} = \frac{\sqrt{35}}{1} = \sqrt{35}$$

16. $\tan 30° = \frac{\sqrt{3}}{3}$

17. $\cot 120° = \frac{1}{\tan 120°} = \frac{1}{-\tan 60°} = \frac{1}{-\sqrt{3}} = -\frac{\sqrt{3}}{3}$

18. $\cos 240° = -\cos 60° = -\frac{1}{2}$

19. $\sec\frac{11\pi}{6} = \frac{1}{\cos\frac{11\pi}{6}} = \frac{1}{\cos\frac{\pi}{6}} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

20. $\sin^2\frac{\pi}{7} + \cos^2\frac{\pi}{7} = 1$

21. $\sin\left(-\frac{2\pi}{3}\right)=\sin\left(-\frac{2\pi}{3}+2\pi\right)$
$$=\sin\frac{4\pi}{3}=-\sin\frac{\pi}{3}$$
$$=-\frac{\sqrt{3}}{2}$$

22. $\csc\left(\frac{22\pi}{3}\right)=\csc\left(\frac{22\pi}{3}-6\pi\right)=\csc\frac{4\pi}{3}$
$$=\frac{1}{\sin\frac{4\pi}{3}}=\frac{1}{-\sin\frac{\pi}{3}}=\frac{1}{-\frac{\sqrt{3}}{2}}$$
$$=-\frac{2}{\sqrt{3}}=-\frac{2\sqrt{3}}{3}$$

23. $\cos 495°=\cos(495°-360°)=\cos 135°$
$$=-\cos 45°=-\frac{\sqrt{2}}{2}$$

24. $\tan\left(-\frac{17\pi}{6}\right)=\tan\left(-\frac{17\pi}{6}+4\pi\right)=\tan\frac{7\pi}{6}$
$$=\tan\frac{\pi}{6}=\frac{\sqrt{3}}{3}$$

25. $\sin^2\frac{\pi}{2}-\cos\pi=(1)^2-(-1)=1+1=2$

26. $\cos\left(\frac{5\pi}{6}+2\pi n\right)+\tan\left(\frac{5\pi}{6}+n\pi\right)$
$$=\cos\frac{5\pi}{6}+\tan\frac{5\pi}{6}=-\cos\frac{\pi}{6}-\tan\frac{\pi}{6}$$
$$=-\frac{\sqrt{3}}{2}-\frac{\sqrt{3}}{3}=-\frac{3\sqrt{3}}{6}-\frac{2\sqrt{3}}{6}$$
$$=-\frac{5\sqrt{3}}{6}$$

27. Begin by converting from degrees to radians.
$$36°=36°\cdot\frac{\pi \text{ radians}}{180°}=\frac{\pi}{5}\text{ radians}$$
$$s=r\theta=40\cdot\frac{\pi}{5}=8\pi\approx 25.13\text{ cm}$$

28. Linear speed is given by $v=r\omega$. It is given that $r=10$ feet and the merry-go-round rotates at 8 revolutions per minute. Convert 8 revolutions per minute to radians per minute.
$$8\text{ revolutions per minute}$$
$$=8\text{ revolutions per minute}\cdot\frac{2\pi\text{ radians}}{1\text{ revolution}}$$
$$=16\pi\text{ radians per minute}$$
$$v=r\omega=(10)(16\pi)=160\pi\approx 502.7\text{ feet per minute}$$
The linear speed of the horse is about 502.7 feet per minute.

29. $\sin\theta=\frac{\text{side opposite }\theta}{\text{hypotenuse}}$
$$\sin 6°=\frac{h}{5280}$$
$$h=5280\sin 6°$$
$$h\approx 551.9\text{ feet}$$

30. $\tan\theta=\frac{\text{side opposite }\theta}{\text{side adjacent }\theta}$
$$\tan\theta=\frac{50}{60}$$
$$\theta=\tan^{-1}\left(\frac{50}{60}\right)$$
$$\theta\approx 40°$$

Section 4.5

Checkpoint Exercises

1. The equation $y=3\sin x$ is of the form $y=A\sin x$ with $A=3$. Thus, the amplitude is $|A|=|3|=3$ The period for both $y=3\sin x$ and $y=\sin x$ is 2π. We find the three x–intercepts, the maximum point, and the minimum point on the interval $[0,2\pi]$ by dividing the period, 2π, by 4, $\frac{\text{period}}{4}=\frac{2\pi}{4}=\frac{\pi}{2}$, then by adding quarter-periods to generate x-values for each of the key points.

The five x-values are

$x = 0$

$x = 0 + \frac{\pi}{2} = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{2} = \pi$

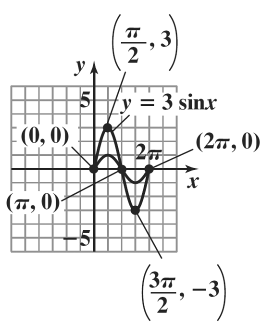
$x = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$

$x = \frac{3\pi}{2} + \frac{\pi}{2} = 2\pi$

Evaluate the function at each value of x.

x	$y = 3\sin x$	coordinates
0	$y = 3\sin 0 = 3 \cdot 0 = 0$	(0, 0)
$\frac{\pi}{2}$	$y = 3\sin\frac{\pi}{2} = 3 \cdot 1 = 3$	$\left(\frac{\pi}{2}, 3\right)$
π	$y = 3\sin x = 3 \cdot 0 = 0$	$(\pi, 0)$
$\frac{3\pi}{2}$	$y = 3\sin\frac{3\pi}{2}$ $= 3(-1) = -3$	$\left(\frac{3\pi}{2}, -3\right)$
2π	$y = 3\sin 2\pi = 3 \cdot 0 = 0$	$(2\pi, 0)$

Connect the five points with a smooth curve and graph one complete cycle of the given function with the graph of $y = \sin x$.



2. The equation $y = -\frac{1}{2}\sin x$ is of the form $y = A\sin x$ with $A = -\frac{1}{2}$. Thus, the amplitude is $|A| = \left|-\frac{1}{2}\right| = \frac{1}{2}$. The period for both $y = -\frac{1}{2}\sin x$ and $y = \sin x$ is 2π.

Find the x–values for the five key points by dividing the period, 2π, by 4, $\frac{\text{period}}{4} = \frac{2\pi}{4} = \frac{\pi}{2}$, then by adding quarter- periods. The five x-values are

$x = 0$

$x = 0 + \frac{\pi}{2} = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{2} = \pi$

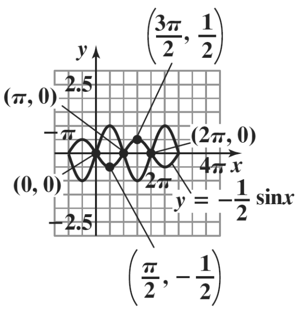
$x = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$

$x = \frac{3\pi}{2} + \frac{\pi}{2} = 2\pi$

Evaluate the function at each value of x.

x	$y = -\frac{1}{2}\sin x$	coordinates
0	$y = -\frac{1}{2}\sin 0$ $= -\frac{1}{2} \cdot 0 = 0$	(0, 0)
$\frac{\pi}{2}$	$y = -\frac{1}{2}\sin\frac{\pi}{2}$ $= -\frac{1}{2} \cdot 1 = -\frac{1}{2}$	$\left(\frac{\pi}{2}, -\frac{1}{2}\right)$
π	$y = -\frac{1}{2}\sin\pi$ $= -\frac{1}{2} \cdot 0 = 0$	$(\pi, 0)$
$\frac{3\pi}{2}$	$y = -\frac{1}{2}\sin\frac{3\pi}{2}$ $= -\frac{1}{2}(-1) = \frac{1}{2}$	$\left(\frac{3\pi}{2}, \frac{1}{2}\right)$
2π	$y = -\frac{1}{2}\sin 2\pi$ $= -\frac{1}{2} \cdot 0 = 0$	$(2\pi, 0)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function with the graph of $y = \sin x$. Extend the pattern of each graph to the left and right as desired.



3. The equation $y = 2\sin\frac{1}{2}x$ is of the form

$y = A\sin Bx$ with $A = 2$ and $B = \frac{1}{2}$.

The amplitude is $|A| = |2| = 2$.

The period is $\frac{2\pi}{B} = \frac{2\pi}{\frac{1}{2}} = 4\pi$.

Find the x–values for the five key points by dividing the period, 4π, by 4, $\frac{\text{period}}{4} = \frac{4\pi}{4} = \pi$, then by adding quarter-periods.

The five x-values are

$x = 0$

$x = 0 + \pi = \pi$

$x = \pi + \pi = 2\pi$

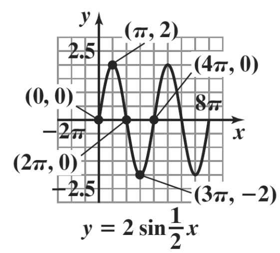
$x = 2\pi + \pi = 3\pi$

$x = 3\pi + \pi = 4\pi$

Evaluate the function at each value of x.

x	$y = 2\sin\frac{1}{2}x$	coordinates
0	$y = 2\sin\left(\frac{1}{2}\cdot 0\right)$ $= 2\sin 0$ $= 2\cdot 0 = 0$	(0, 0)
π	$y = 2\sin\left(\frac{1}{2}\cdot \pi\right)$ $= 2\sin\frac{\pi}{2} = 2\cdot 1 = 2$	$(\pi, 2)$
2π	$y = 2\sin\left(\frac{1}{2}\cdot 2\pi\right)$ $= 2\sin\pi = 2\cdot 0 = 0$	$(2\pi, 0)$
3π	$y = 2\sin\left(\frac{1}{2}\cdot 3\pi\right)$ $= 2\sin\frac{3\pi}{2}$ $= 2\cdot(-1) = -2$	$(3\pi, -2)$
4π	$y = 2\sin\left(\frac{1}{2}\cdot 4\pi\right)$ $= 2\sin 2\pi = 2\cdot 0 = 0$	$(4\pi, 0)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function. Extend the pattern of the graph another full period to the right.



4. The equation $y = 3\sin\left(2x - \frac{\pi}{3}\right)$ is of the form

$y = A\sin(Bx - C)$ with $A = 3$, $B = 2$, and $C = \frac{\pi}{3}$. The amplitude is $|A| = |3| = 3$.

The period is $\frac{2\pi}{B} = \frac{2\pi}{2} = \pi$.

The phase shift is $\frac{C}{B} = \frac{\frac{\pi}{3}}{2} = \frac{\pi}{3}\cdot\frac{1}{2} = \frac{\pi}{6}$.

Find the x-values for the five key points by dividing the period, π, by 4, $\frac{\text{period}}{4} = \frac{\pi}{4}$, then by adding quarter-periods to the value of x where the cycle begins, $x = \frac{\pi}{6}$.

The five x-values are

$x = \frac{\pi}{6}$

$x = \frac{\pi}{6} + \frac{\pi}{4} = \frac{2\pi}{12} + \frac{3\pi}{12} = \frac{5\pi}{12}$

$x = \frac{5\pi}{12} + \frac{\pi}{4} = \frac{5\pi}{12} + \frac{3\pi}{12} = \frac{8\pi}{12} = \frac{2\pi}{3}$

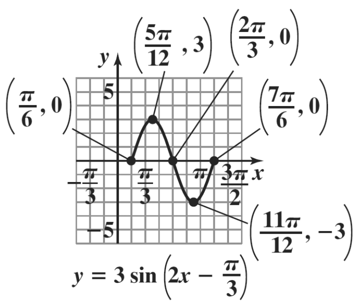
$x = \frac{2\pi}{3} + \frac{\pi}{4} = \frac{8\pi}{12} + \frac{3\pi}{12} = \frac{11\pi}{12}$

$x = \frac{11\pi}{12} + \frac{\pi}{4} = \frac{11\pi}{12} + \frac{3\pi}{12} = \frac{14\pi}{12} = \frac{7\pi}{6}$

Evaluate the function at each value of x.

x	$y=3\sin\left(2x-\frac{\pi}{3}\right)$	coordinates
$\frac{\pi}{6}$	$y=3\sin\left(2\cdot\frac{\pi}{6}-\frac{\pi}{3}\right)$ $=3\sin 0=3\cdot 0=0$	$\left(\frac{\pi}{6},0\right)$
$\frac{5\pi}{12}$	$y=3\sin\left(2\cdot\frac{5\pi}{12}-\frac{\pi}{3}\right)$ $=3\sin\frac{3\pi}{6}=3\sin\frac{\pi}{2}$ $=3\cdot 1=3$	$\left(\frac{5\pi}{12},3\right)$
$\frac{2\pi}{3}$	$y=3\sin\left(2\cdot\frac{2\pi}{3}-\frac{\pi}{3}\right)$ $=3\sin\frac{3\pi}{3}=3\sin\pi$ $=3\cdot 0=0$	$\left(\frac{2\pi}{3},0\right)$
$\frac{11\pi}{12}$	$y=3\sin\left(2\cdot\frac{11\pi}{12}-\frac{\pi}{3}\right)$ $=3\sin\frac{9\pi}{6}=3\sin\frac{3\pi}{2}$ $=3(-1)=-3$	$\left(\frac{11\pi}{12},-3\right)$
$\frac{7\pi}{6}$	$y=3\sin\left(2\cdot\frac{7\pi}{6}-\frac{\pi}{3}\right)$ $=3\sin\frac{6\pi}{3}=3\sin 2\pi$ $=3\cdot 0=0$	$\left(\frac{7\pi}{6},0\right)$

Connect the five key points with a smooth curve and graph one complete cycle of the given graph.



5. The equation $y=-4\cos\pi x$ is of the form $y=A\cos Bx$ with $A=-4$, and $B=\pi$.
Thus, the amplitude is $|A|=|-4|=4$.
The period is $\frac{2\pi}{B}=\frac{2\pi}{\pi}=2$.

Find the x-values for the five key points by dividing the period, 2, by 4, $\frac{\text{period}}{4}=\frac{2}{4}=\frac{1}{2}$, then by adding quarter periods to the value of x where the cycle begins. The five x-values are

$x=0$

$x=0+\frac{1}{2}=\frac{1}{2}$

$x=\frac{1}{2}+\frac{1}{2}=1$

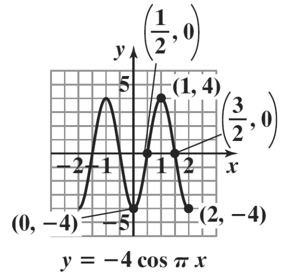
$x=1+\frac{1}{2}=\frac{3}{2}$

$x=\frac{3}{2}+\frac{1}{2}=2$

Evaluate the function at each value of x.

x	$y=-4\cos\pi x$	coordinates
0	$y=-4\cos(\pi\cdot 0)$ $=-4\cos 0=-4$	(0, –4)
$\frac{1}{2}$	$y=-4\cos\left(\pi\cdot\frac{1}{2}\right)$ $=-4\cos\frac{\pi}{2}=0$	$\left(\frac{1}{2},0\right)$
1	$y=-4\cos(\pi\cdot 1)$ $=-4\cos\pi=4$	(1, 4)
$\frac{3}{2}$	$y=-4\cos\left(\pi\cdot\frac{3}{2}\right)$ $=-4\cos\frac{3\pi}{2}=0$	$\left(\frac{3}{2},0\right)$
2	$y=-4\cos(\pi\cdot 2)$ $=-4\cos 2\pi=-4$	(2, –4)

Connect the five key points with a smooth curve and graph one complete cycle of the given function. Extend the pattern of the graph another full period to the left.



6. $y = \frac{3}{2}\cos(2x+\pi) = \frac{3}{2}\cos(2x-(-\pi))$

The equation is of the form $y = A\cos(Bx-C)$ with $A = \frac{3}{2}$, $B = 2$, and $C = -\pi$.

Thus, the amplitude is $|A| = \left|\frac{3}{2}\right| = \frac{3}{2}$.

The period is $\frac{2\pi}{B} = \frac{2\pi}{2} = \pi$.

The phase shift is $\frac{C}{B} = \frac{-\pi}{2} = -\frac{\pi}{2}$.

Find the x-values for the five key points by dividing the period, π, by 4, $\frac{\text{period}}{4} = \frac{\pi}{4}$, then by adding quarter-periods to the value of x where the cycle begins, $x = -\frac{\pi}{2}$.

The five x-values are

$x = -\frac{\pi}{2}$

$x = -\frac{\pi}{2} + \frac{\pi}{4} = -\frac{\pi}{4}$

$x = -\frac{\pi}{4} + \frac{\pi}{4} = 0$

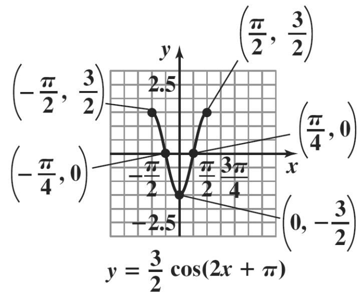
$x = 0 + \frac{\pi}{4} = \frac{\pi}{4}$

$x = \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$

Evaluate the function at each value of x.

x	$y = \frac{3}{2}\cos(2x+\pi)$	coordinates
$-\frac{\pi}{2}$	$y = \frac{3}{2}\cos(-\pi+\pi)$ $= \frac{3}{2}\cdot 1 = \frac{3}{2}$	$\left(-\frac{\pi}{2}, \frac{3}{2}\right)$
$-\frac{\pi}{4}$	$y = \frac{3}{2}\cos\left(-\frac{\pi}{2}+\pi\right)$ $= \frac{3}{2}\cdot 0 = 0$	$\left(-\frac{\pi}{4}, 0\right)$
0	$y = \frac{3}{2}\cos(0+\pi)$ $= \frac{3}{2}\cdot -1 = -\frac{3}{2}$	$\left(0, -\frac{3}{2}\right)$
$\frac{\pi}{4}$	$y = \frac{3}{2}\cos\left(\frac{\pi}{2}+\pi\right)$ $= \frac{3}{2}\cdot 0 = 0$	$\left(\frac{\pi}{4}, 0\right)$
$\frac{\pi}{2}$	$y = \frac{3}{2}\cos(\pi+\pi)$ $= \frac{3}{2}\cdot 1 = \frac{3}{2}$	$\left(\frac{\pi}{2}, \frac{3}{2}\right)$

Connect the five key points with a smooth curve and graph one complete cycle of the given graph.



7. The graph of $y = 2\cos x + 1$ is the graph of $y = 2\cos x$ shifted one unit upwards. The period for both functions is 2π. The quarter-period is $\frac{2\pi}{4}$ or $\frac{\pi}{2}$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{\pi}{2} = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{2} = \pi$

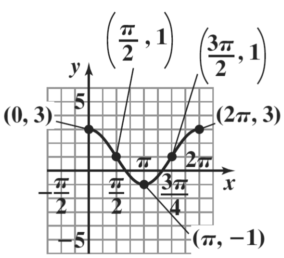
$x = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$

$x = \frac{3\pi}{2} + \frac{\pi}{2} = 2\pi$

Evaluate the function at each value of x.

x	$y = 2\cos x + 1$	coordinates
0	$y = 2\cos 0 + 1$ $= 2 \cdot 1 + 1 = 3$	$(0, 3)$
$\frac{\pi}{2}$	$y = 2\cos\frac{\pi}{2} + 1$ $= 2 \cdot 0 + 1 = 1$	$\left(\frac{\pi}{2}, 1\right)$
π	$y = 2\cos\pi + 1$ $= 2 \cdot (-1) + 1 = -1$	$(\pi, -1)$
$\frac{3\pi}{2}$	$y = 2\cos\frac{3\pi}{2} + 1$ $= 2 \cdot 0 + 1 = 1$	$\left(\frac{3\pi}{2}, 1\right)$
2π	$y = 2\cos 2\pi + 1$ $= 2 \cdot 1 + 1 = 3$	$(2\pi, 3)$

By connecting the points with a smooth curve, we obtain one period of the graph.



$y = 2\cos x + 1$

8. A, the amplitude, is the maximum value of y. The graph shows that this maximum value is 4, Thus, $A = 4$. The period is $\frac{\pi}{2}$, and period $= \frac{2\pi}{B}$. Thus,

$$\frac{\pi}{2} = \frac{2\pi}{B}$$
$$\pi B = 4\pi$$
$$B = 4$$

Substitute these values into $y = A\sin Bx$.

The graph is modeled by $y = 4\sin 4x$.

9. Because the hours of daylight ranges from a minimum of 10 hours to a maximum of 14 hours, the curve oscillates about the middle value, 12 hours. Thus, D = 12. The maximum number of hours is 2 hours above 12 hours. Thus, A = 2. The graph shows that one complete cycle occurs in 12–0, or 12 months. The period is 12. Thus,

$$12 = \frac{2\pi}{B}$$
$$12B = 2\pi$$
$$B = \frac{2\pi}{12} = \frac{\pi}{6}$$

The graph shows that the starting point of the cycle is shifted from 0 to 3. The phase shift, $\frac{C}{B}$, is 3.

$$3 = \frac{C}{B}$$
$$3 = \frac{C}{\frac{\pi}{6}}$$
$$\frac{\pi}{2} = C$$

Substitute these values into $y = A\sin(Bx - C) + D$.

The number of hours of daylight is modeled by

$$y = 2\sin\left(\frac{\pi}{6}x - \frac{\pi}{2}\right) + 12.$$

Concept and Vocabulary Check 4.5

1. $|A|$; $\frac{2\pi}{B}$

2. 3; 4π

3. π; 0; $\frac{\pi}{4}$; $\frac{\pi}{2}$; $\frac{3\pi}{4}$; π

4. $\frac{C}{B}$; right; left

5. $|A|$; $\frac{2\pi}{B}$

6. $\frac{1}{2}$; $\frac{2\pi}{3}$

7. false

8. true

9. true

10. true