Exercise Set 4.5

1. The equation $y = 4\sin x$ is of the form $y = A\sin x$ with $A = 4$. Thus, the amplitude is $|A| = |4| = 4$.

The period is 2π. The quarter-period is $\frac{2\pi}{4}$ or $\frac{\pi}{2}$.

The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{\pi}{2} = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{2} = \pi$

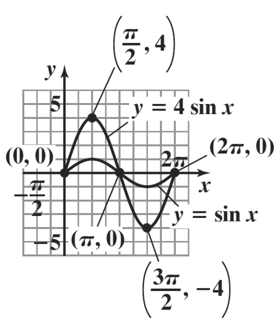
$x = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$

$x = \frac{3\pi}{2} + \frac{\pi}{2} = 2\pi$

Evaluate the function at each value of x.

x	$y = 4\sin x$	coordinates
0	$y = 4\sin 0 = 4\cdot 0 = 0$	(0, 0)
$\frac{\pi}{2}$	$y = 4\sin\frac{\pi}{2} = 4\cdot 1 = 4$	$\left(\frac{\pi}{2}, 4\right)$
π	$y = 4\sin\pi = 4\cdot 0 = 0$	$(\pi, 0)$
$\frac{3\pi}{2}$	$y = 4\sin\frac{3\pi}{2}$ $= 4(-1) = -4$	$\left(\frac{3\pi}{2}, -4\right)$
2π	$y = 4\sin 2\pi = 4\cdot 0 = 0$	$(2\pi, 0)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function with the graph of $y = \sin x$.



2. The equation $y = 5\sin x$ is of the form $y = A\sin x$ with $A = 5$. Thus, the amplitude is $|A| = |5| = 5$.

The period is 2π. The quarter-period is $\frac{2\pi}{4}$ or $\frac{\pi}{2}$.

The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{\pi}{2} = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{2} = \pi$

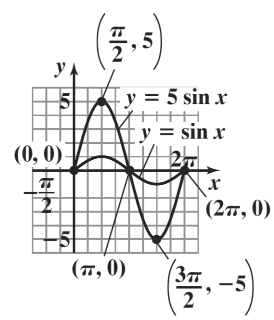
$x = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$

$x = \frac{3\pi}{2} + \frac{\pi}{2} = 2\pi$

Evaluate the function at each value of x.

x	$y = 5\sin x$	coordinates
0	$y = 5\sin 0 = 5\cdot 0 = 0$	(0, 0)
$\frac{\pi}{2}$	$y = 5\sin\frac{\pi}{2} = 5\cdot 1 = 5$	$\left(\frac{\pi}{2}, 5\right)$
π	$y = 5\sin\pi = 5\cdot 0 = 0$	$(\pi, 0)$
$\frac{3\pi}{2}$	$y = 5\sin\frac{3\pi}{2} = 5(-1) = -5$	$\left(\frac{3\pi}{2}, -5\right)$
2π	$y = 5\sin 2\pi = 5\cdot 0 = 0$	$(2\pi, 0)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function with the graph of $y = \sin x$.



3. The equation $y = \frac{1}{3}\sin x$ is of the form $y = A\sin x$ with $A = \frac{1}{3}$. Thus, the amplitude is $|A| = \left|\frac{1}{3}\right| = \frac{1}{3}$.
The period is 2π. The quarter-period is $\frac{2\pi}{4}$ or $\frac{\pi}{2}$.
The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{\pi}{2} = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{2} = \pi$

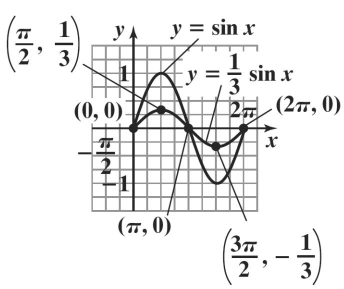
$x = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$

$x = \frac{3\pi}{2} + \frac{\pi}{2} = 2\pi$

Evaluate the function at each value of x.

x	$y = \frac{1}{3}\sin x$	coordinates
0	$y = \frac{1}{3}\sin 0 = \frac{1}{3}\cdot 0 = 0$	(0, 0)
$\frac{\pi}{2}$	$y = \frac{1}{3}\sin\frac{\pi}{2} = \frac{1}{3}\cdot 1 = \frac{1}{3}$	$\left(\frac{\pi}{2}, \frac{1}{3}\right)$
π	$y = \frac{1}{3}\sin\pi = \frac{1}{3}\cdot 0 = 0$	$(\pi, 0)$
$\frac{3\pi}{2}$	$y = \frac{1}{3}\sin\frac{3\pi}{2} = \frac{1}{3}(-1) = -\frac{1}{3}$	$\left(\frac{3\pi}{2}, -\frac{1}{3}\right)$
2π	$y = \frac{1}{3}\sin 2\pi = \frac{1}{3}\cdot 0 = 0$	$(2\pi, 0)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function with the graph of $y = \sin x$.



4. The equation $y = \frac{1}{4}\sin x$ is of the form $y = A\sin x$ with $A = \frac{1}{4}$. Thus, the amplitude is $|A| = \left|\frac{1}{4}\right| = \frac{1}{4}$.
The period is 2π. The quarter-period is $\frac{2\pi}{4}$ or $\frac{\pi}{2}$.
The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{\pi}{2} = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{2} = \pi$

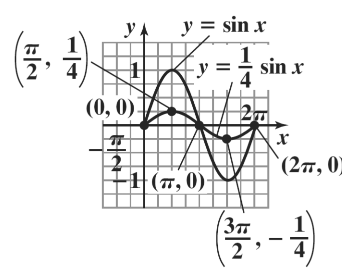
$x = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$

$x = \frac{3\pi}{2} + \frac{\pi}{2} = 2\pi$

Evaluate the function at each value of x.

x	$y = \frac{1}{4}\sin x$	coordinates
0	$y = \frac{1}{4}\sin 0 = \frac{1}{4}\cdot 0 = 0$	(0, 0)
$\frac{\pi}{2}$	$y = \frac{1}{4}\sin\frac{\pi}{2} = \frac{1}{4}\cdot 1 = \frac{1}{4}$	$\left(\frac{\pi}{2}, \frac{1}{4}\right)$
π	$y = \frac{1}{4}\sin\pi = \frac{1}{4}\cdot 0 = 0$	$(\pi, 0)$
$\frac{3\pi}{2}$	$y = \frac{1}{4}\sin\frac{3\pi}{2} = \frac{1}{4}(-1) = -\frac{1}{4}$	$\left(\frac{3\pi}{2}, -\frac{1}{4}\right)$
2π	$y = \frac{1}{4}\sin 2\pi = \frac{1}{4}\cdot 0 = 0$	$(2\pi, 0)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function with the graph of $y = \sin x$.



5. The equation $y = -3\sin x$ is of the form $y = A\sin x$ with $A = -3$. Thus, the amplitude is $|A| = |-3| = 3$.

The period is 2π. The quarter-period is $\frac{2\pi}{4}$ or $\frac{\pi}{2}$.

The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{\pi}{2} = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{2} = \pi$

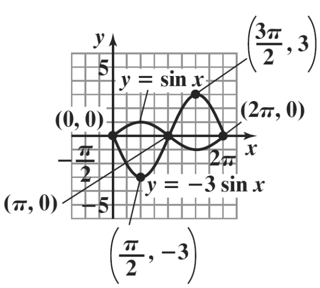
$x = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$

$x = \frac{3\pi}{2} + \frac{\pi}{2} = 2\pi$

Evaluate the function at each value of x.

x	$y = -3\sin x$	coordinates
0	$y = -3\sin x$ $= -3 \cdot 0 = 0$	(0, 0)
$\frac{\pi}{2}$	$y = -3\sin\frac{\pi}{2}$ $= -3 \cdot 1 = -3$	$\left(\frac{\pi}{2}, -3\right)$
π	$y = -3\sin\pi$ $= -3 \cdot 0 = 0$	$(\pi, 0)$
$\frac{3\pi}{2}$	$y = -3\sin\frac{3\pi}{2}$ $= -3(-1) = 3$	$\left(\frac{3\pi}{2}, 3\right)$
2π	$y = -3\sin 2\pi$ $= -3 \cdot 0 = 0$	$(2\pi, 0)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function with the graph of $y = \sin x$.



6. The equation $y = -4\sin x$ is of the form $y = A\sin x$ with $A = -4$. Thus, the amplitude is $|A| = |-4| = 4$.

The period is 2π. The quarter-period is $\frac{2\pi}{4}$ or $\frac{\pi}{2}$.

The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{\pi}{2} = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{2} = \pi$

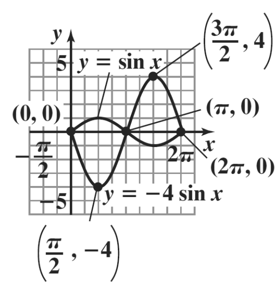
$x = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$

$x = \frac{3\pi}{2} + \frac{\pi}{2} = 2\pi$

Evaluate the function at each value of x.

x	$y = -4\sin x$	coordinates
0	$y = -4\sin 0 = -4 \cdot 0 = 0$	(0, 0)
$\frac{\pi}{2}$	$y = -4\sin\frac{\pi}{2} = -4 \cdot 1 = -4$	$\left(\frac{\pi}{2}, -4\right)$
π	$y = -4\sin\pi = -4 \cdot 0 = 0$	$(\pi, 0)$
$\frac{3\pi}{2}$	$y = -4\sin\frac{3\pi}{2} = -4(-1) = 4$	$\left(\frac{3\pi}{2}, 4\right)$
2π	$y = -4\sin 2\pi = -4 \cdot 0 = 0$	$(2\pi, 0)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function with the graph of $y = \sin x$.



7. The equation $y = \sin 2x$ is of the form $y = A\sin Bx$ with $A = 1$ and $B = 2$. The amplitude is $|A| = |1| = 1$. The period is $\frac{2\pi}{B} = \frac{2\pi}{2} = \pi$. The quarter-period is $\frac{\pi}{4}$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{\pi}{4}$

$x = \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$

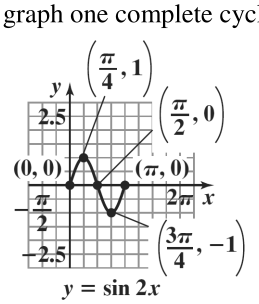
$x = \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$

$x = \frac{3\pi}{4} + \frac{\pi}{4} = \pi$

Evaluate the function at each value of x.

x	$y = \sin 2x$	coordinates
0	$y = \sin 2\cdot 0 = \sin 0 = 0$	(0, 0)
$\frac{\pi}{4}$	$y = \sin\left(2\cdot\frac{\pi}{4}\right)$ $= \sin\frac{\pi}{2} = 1$	$\left(\frac{\pi}{4}, 1\right)$
$\frac{\pi}{2}$	$y = \sin\left(2\cdot\frac{\pi}{2}\right)$ $= \sin\pi = 0$	$\left(\frac{\pi}{2}, 0\right)$
$\frac{3\pi}{4}$	$y = \sin\left(2\cdot\frac{3\pi}{4}\right)$ $= \sin\frac{3\pi}{2} = -1$	$\left(\frac{3\pi}{4}, -1\right)$
π	$y = \sin(2\cdot\pi)$ $= \sin 2\pi = 0$	$(\pi, 0)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



8. The equation $y = \sin 4x$ is of the form $y = A\sin Bx$ with $A = 1$ and $B = 4$. Thus, the amplitude is $|A| = |1| = 1$. The period is $\frac{2\pi}{B} = \frac{2\pi}{4} = \frac{\pi}{2}$. The quarter-period is $\frac{\frac{\pi}{2}}{4} = \frac{\pi}{2}\cdot\frac{1}{4} = \frac{\pi}{8}$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{\pi}{8} = \frac{\pi}{8}$

$x = \frac{\pi}{8} + \frac{\pi}{8} = \frac{\pi}{4}$

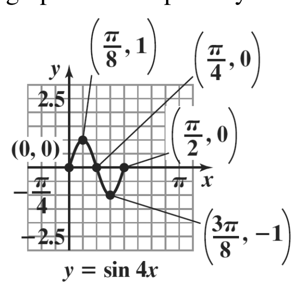
$x = \frac{\pi}{4} + \frac{\pi}{8} = \frac{3\pi}{8}$

$x = \frac{3\pi}{8} + \frac{\pi}{8} = \frac{\pi}{2}$

Evaluate the function at each value of x.

x	$y = \sin 4x$	coordinates
0	$y = \sin(4\cdot 0) = \sin 0 = 0$	(0, 0)
$\frac{\pi}{8}$	$y = \sin\left(4\cdot\frac{\pi}{8}\right) = \sin\frac{\pi}{2} = 1$	$\left(\frac{\pi}{8}, 1\right)$
$\frac{\pi}{4}$	$y = \sin\left(4\cdot\frac{\pi}{4}\right) = \sin\pi = 0$	$\left(\frac{\pi}{4}, 0\right)$
$\frac{3\pi}{8}$	$y = \sin\left(4\cdot\frac{3\pi}{8}\right)$ $= \sin\frac{3\pi}{2} = -1$	$\left(\frac{3\pi}{8}, -1\right)$
$\frac{\pi}{2}$	$y = \sin 2\pi = 0$	$\left(\frac{\pi}{2}, 0\right)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



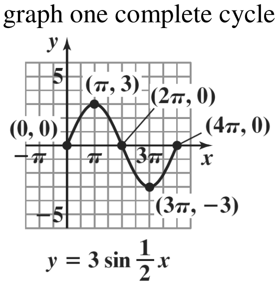
9. The equation $y = 3\sin\frac{1}{2}x$ is of the form $y = A\sin Bx$ with $A = 3$ and $B = \frac{1}{2}$. The amplitude is $|A| = |3| = 3$. The period is $\frac{2\pi}{B} = \frac{2\pi}{\frac{1}{2}} = 2\pi \cdot 2 = 4\pi$. The quarter-period is $\frac{4\pi}{4} = \pi$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$
$x = 0 + \pi = \pi$
$x = \pi + \pi = 2\pi$
$x = 2\pi + \pi = 3\pi$
$x = 3\pi + \pi = 4\pi$

Evaluate the function at each value of x.

x	$y = 3\sin\frac{1}{2}x$	coordinates
0	$y = 3\sin\left(\frac{1}{2}\cdot 0\right)$ $= 3\sin 0 = 3\cdot 0 = 0$	(0, 0)
π	$y = 3\sin\left(\frac{1}{2}\cdot \pi\right)$ $= 3\sin\frac{\pi}{2} = 3\cdot 1 = 3$	$(\pi, 3)$
2π	$y = 3\sin\left(\frac{1}{2}\cdot 2\pi\right)$ $= 3\sin\pi = 3\cdot 0 = 0$	$(2\pi, 0)$
3π	$y = 3\sin\left(\frac{1}{2}\cdot 3\pi\right)$ $= 3\sin\frac{3\pi}{2}$ $= 3(-1) = -3$	$(3\pi, -3)$
4π	$y = 3\sin\left(\frac{1}{2}\cdot 4\pi\right)$ $= 3\sin 2\pi = 3\cdot 0 = 0$	$(4\pi, 0)$

Connect the five points with a smooth curve and graph one complete cycle of the given function.



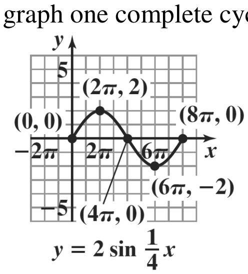
10. The equation $y = 2\sin\frac{1}{4}x$ is of the form $y = A\sin Bx$ with $A = 2$ and $B = \frac{1}{4}$. Thus, the amplitude is $|A| = |2| = 2$. The period is $\frac{2\pi}{B} = \frac{2\pi}{\frac{1}{4}} = 2\pi \cdot 4 = 8\pi$. The quarter-period is $\frac{8\pi}{4} = 2\pi$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$
$x = 0 + 2\pi = 2\pi$
$x = 2\pi + 2\pi = 4\pi$
$x = 4\pi + 2\pi = 6\pi$
$x = 6\pi + 2\pi = 8\pi$

Evaluate the function at each value of x.

x	$y = 2\sin\frac{1}{4}x$	coordinates
0	$y = 2\sin\left(\frac{1}{4}\cdot 0\right)$ $= 2\sin 0 = 2\cdot 0 = 0$	(0, 0)
2π	$y = 2\sin\left(\frac{1}{4}\cdot 2\pi\right)$ $= 2\sin\frac{\pi}{2} = 2\cdot 1 = 2$	$(2\pi, 2)$
4π	$y = 2\sin\pi = 2\cdot 0 = 0$	$(4\pi, 0)$
6π	$y = 2\sin\frac{3\pi}{2} = 2(-1) = -2$	$(6\pi, -2)$
8π	$y = 2\sin 2\pi = 2\cdot 0 = 0$	$(8\pi, 0)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



11. The equation $y = 4\sin \pi x$ is of the form $y = A\sin Bx$ with $A = 4$ and $B = \pi$. The amplitude is $|A| = |4| = 4$. The period is $\frac{2\pi}{B} = \frac{2\pi}{\pi} = 2$. The quarter-period is $\frac{2}{4} = \frac{1}{2}$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{1}{2} = \frac{1}{2}$

$x = \frac{1}{2} + \frac{1}{2} = 1$

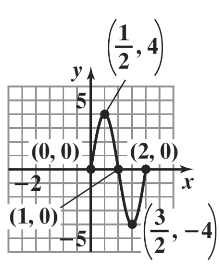
$x = 1 + \frac{1}{2} = \frac{3}{2}$

$x = \frac{3}{2} + \frac{1}{2} = 2$

Evaluate the function at each value of x.

x	$y = 4\sin \pi x$	coordinates
0	$y = 4\sin(\pi \cdot 0)$ $= 4\sin 0 = 4 \cdot 0 = 0$	(0, 0)
$\frac{1}{2}$	$y = 4\sin\left(\pi \cdot \frac{1}{2}\right)$ $= 4\sin\frac{\pi}{2} = 4(1) = 4$	$\left(\frac{1}{2}, 4\right)$
1	$y = 4\sin(\pi \cdot 1)$ $= 4\sin \pi = 4 \cdot 0 = 0$	(1, 0)
$\frac{3}{2}$	$y = 4\sin\left(\pi \cdot \frac{3}{2}\right)$ $= 4\sin\frac{3\pi}{2}$ $= 4(-1) = -4$	$\left(\frac{3}{2}, -4\right)$
2	$y = 4\sin(\pi \cdot 2)$ $= 4\sin 2\pi = 4 \cdot 0 = 0$	(2, 0)

Connect the five points with a smooth curve and graph one complete cycle of the given function.



$y = 4 \sin \pi x$

12. The equation $y = 3\sin 2\pi x$ is of the form $y = A\sin Bx$ with $A = 3$ and $B = 2\pi$. The amplitude is $|A| = |3| = 3$. The period is $\frac{2\pi}{B} = \frac{2\pi}{2\pi} = 1$. The quarter-period is $\frac{1}{4}$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{1}{4} = \frac{1}{4}$

$x = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

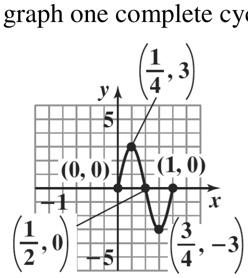
$x = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$

$x = \frac{3}{4} + \frac{1}{4} = 1$

Evaluate the function at each value of x.

x	$y = 3\sin 2\pi x$	coordinates
0	$y = 3\sin(2\pi \cdot 0)$ $= 3\sin 0 = 3 \cdot 0 = 0$	(0, 0)
$\frac{1}{4}$	$y = 3\sin\left(2\pi \cdot \frac{1}{4}\right)$ $= 3\sin\frac{\pi}{2} = 3 \cdot 1 = 3$	$\left(\frac{1}{4}, 3\right)$
$\frac{1}{2}$	$y = 3\sin\left(2\pi \cdot \frac{1}{2}\right)$ $= 3\sin \pi = 3 \cdot 0 = 0$	$\left(\frac{1}{2}, 0\right)$
$\frac{3}{4}$	$y = 3\sin\left(2\pi \cdot \frac{3}{4}\right)$ $= 3\sin\frac{3\pi}{2} = 3(-1) = -3$	$\left(\frac{3}{4}, -3\right)$
1	$y = 3\sin(2\pi \cdot 1)$ $= 3\sin 2\pi = 3 \cdot 0 = 0$	(1, 0)

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



$y = 3 \sin 2\pi x$

13. The equation $y = -3\sin 2\pi x$ is of the form $y = A\sin Bx$ with $A = -3$ and $B = 2\pi$. The amplitude is $|A| = |-3| = 3$. The period is $\frac{2\pi}{B} = \frac{2\pi}{2\pi} = 1$. The quarter-period is $\frac{1}{4}$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{1}{4} = \frac{1}{4}$

$x = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

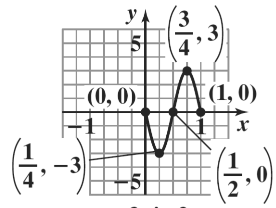
$x = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$

$x = \frac{3}{4} + \frac{1}{4} = 1$

Evaluate the function at each value of x.

x	$y = -3\sin 2\pi x$	coordinates
0	$y = -3\sin(2\pi \cdot 0)$ $= -3\sin 0$ $= -3 \cdot 0 = 0$	$(0, 0)$
$\frac{1}{4}$	$y = -3\sin\left(2\pi \cdot \frac{1}{4}\right)$ $= -3\sin\frac{\pi}{2}$ $= -3 \cdot 1 = -3$	$\left(\frac{1}{4}, -3\right)$
$\frac{1}{2}$	$y = -3\sin\left(2\pi \cdot \frac{1}{2}\right)$ $= -3\sin \pi$ $= -3 \cdot 0 = 0$	$\left(\frac{1}{2}, 0\right)$
$\frac{3}{4}$	$y = -3\sin\left(2\pi \cdot \frac{3}{4}\right)$ $= -3\sin\frac{3\pi}{2}$ $= -3(-1) = 3$	$\left(\frac{3}{4}, 3\right)$
1	$y = -3\sin(2\pi \cdot 1)$ $= -3\sin 2\pi$ $= -3 \cdot 0 = 0$	$(1, 0)$

Connect the five points with a smooth curve and graph one complete cycle of the given function.



14. The equation $y = -2\sin \pi x$ is of the form $y = A\sin Bx$ with $A = -2$ and $B = \pi$. The amplitude is $|A| = |-2| = 2$. The period is $\frac{2\pi}{B} = \frac{2\pi}{\pi} = 2$. The quarter-period is $\frac{2}{4} = \frac{1}{2}$.

The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{1}{2} = \frac{1}{2}$

$x = \frac{1}{2} + \frac{1}{2} = 1$

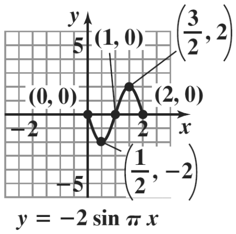
$x = 1 + \frac{1}{2} = \frac{3}{2}$

$x = \frac{3}{2} + \frac{1}{2} = 2$

Evaluate the function at each value of x.

x	$y = -2\sin \pi x$	coordinates
0	$y = -2\sin(\pi \cdot 0)$ $= -2\sin 0 = -2 \cdot 0 = 0$	$(0, 0)$
$\frac{1}{2}$	$y = -2\sin\left(\pi \cdot \frac{1}{2}\right)$ $= -2\sin\frac{\pi}{2} = -2 \cdot 1 = -2$	$\left(\frac{1}{2}, -2\right)$
1	$y = -2\sin(\pi \cdot 1)$ $= -2\sin \pi = -2 \cdot 0 = 0$	$(1, 0)$
$\frac{3}{2}$	$y = -2\sin\left(\pi \cdot \frac{3}{2}\right)$ $= -2\sin\frac{3\pi}{2} = -2(-1) = 2$	$\left(\frac{3}{2}, 2\right)$
2	$y = -2\sin(\pi \cdot 2)$ $= -2\sin 2\pi = -2 \cdot 0 = 0$	$(2, 0)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



15. The equation $y = -\sin\frac{2}{3}x$ is of the form $y = A\sin Bx$ with $A = -1$ and $B = \frac{2}{3}$.

The amplitude is $|A| = |-1| = 1$.

The period is $\frac{2\pi}{B} = \frac{2\pi}{\frac{2}{3}} = 2\pi \cdot \frac{3}{2} = 3\pi$.

The quarter-period is $\frac{3\pi}{4}$. The cycle begins at $x = 0$.

Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{3\pi}{4} = \frac{3\pi}{4}$

$x = \frac{3\pi}{4} + \frac{3\pi}{4} = \frac{3\pi}{2}$

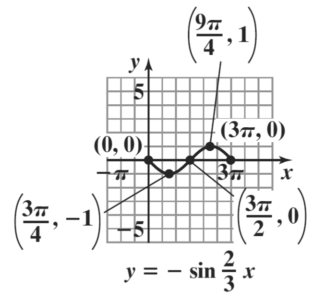
$x = \frac{3\pi}{2} + \frac{3\pi}{4} = \frac{9\pi}{4}$

$x = \frac{9\pi}{4} + \frac{3\pi}{4} = 3\pi$

Evaluate the function at each value of x.

x	$y = -\sin\frac{2}{3}x$	coordinates
0	$y = -\sin\left(\frac{2}{3}\cdot 0\right)$ $= -\sin 0 = 0$	(0, 0)
$\frac{3\pi}{4}$	$y = -\sin\left(\frac{2}{3}\cdot\frac{3\pi}{4}\right)$ $= -\sin\frac{\pi}{2} = -1$	$\left(\frac{3\pi}{4}, -1\right)$
$\frac{3\pi}{2}$	$y = -\sin\left(\frac{2}{3}\cdot\frac{3\pi}{2}\right)$ $= -\sin\pi = 0$	$\left(\frac{3\pi}{2}, 0\right)$
$\frac{9\pi}{4}$	$y = -\sin\left(\frac{2}{3}\cdot\frac{9\pi}{4}\right)$ $= -\sin\frac{3\pi}{2}$ $= -(-1) = 1$	$\left(\frac{9\pi}{4}, 1\right)$
3π	$y = -\sin\left(\frac{2}{3}\cdot 3\pi\right)$ $= -\sin 2\pi = 0$	$(3\pi, 0)$

Connect the five points with a smooth curve and graph one complete cycle of the given function.



16. The equation $y = -\sin\frac{4}{3}x$ is of the form $y = A\sin Bx$ with $A = -1$ and $B = \frac{4}{3}$.

The amplitude is $|A| = |-1| = 1$.

The period is $\frac{2\pi}{B} = \frac{2\pi}{\frac{4}{3}} = 2\pi \cdot \frac{3}{4} = \frac{3\pi}{2}$.

The quarter-period is $\frac{\frac{3\pi}{2}}{4} = \frac{3\pi}{2}\cdot\frac{1}{4} = \frac{3\pi}{8}$.

The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{3\pi}{8} = \frac{3\pi}{8}$

$x = \frac{3\pi}{8} + \frac{3\pi}{8} = \frac{3\pi}{4}$

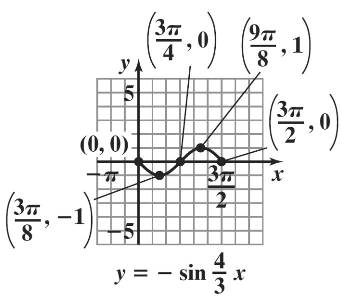
$x = \frac{3\pi}{4} + \frac{3\pi}{8} = \frac{9\pi}{8}$

$x = \frac{9\pi}{8} + \frac{3\pi}{8} = \frac{3\pi}{2}$

Evaluate the function at each value of x.

x	$y = -\sin\frac{4}{3}x$	coordinates
0	$y = -\sin\left(\frac{4}{3}\cdot 0\right)$ $= -\sin 0 = 0$	$(0, 0)$
$\frac{3\pi}{8}$	$y = -\sin\left(\frac{4}{3}\cdot\frac{3\pi}{8}\right)$ $= -\sin\frac{\pi}{2} = -1$	$\left(\frac{3\pi}{8}, -1\right)$
$\frac{3\pi}{4}$	$y = -\sin\left(\frac{4}{3}\cdot\frac{3\pi}{4}\right)$ $= -\sin\pi = 0$	$\left(\frac{3\pi}{4}, 0\right)$
$\frac{9\pi}{8}$	$y = -\sin\left(\frac{4}{3}\cdot\frac{9\pi}{8}\right)$ $= -\sin\frac{3\pi}{2} = -(-1) = 1$	$\left(\frac{9\pi}{8}, 1\right)$
$\frac{3\pi}{2}$	$y = -\sin\left(\frac{4}{3}\cdot\frac{3\pi}{2}\right)$ $= -\sin 2\pi = 0$	$\left(\frac{3\pi}{2}, 0\right)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



17. The equation $y = \sin(x - \pi)$ is of the form $y = A\sin(Bx - C)$ with $A = 1$, $B = 1$, and $C = \pi$. The amplitude is $|A| = |1| = 1$. The period is $\frac{2\pi}{B} = \frac{2\pi}{1} = 2\pi$. The phase shift is $\frac{C}{B} = \frac{\pi}{1} = \pi$. The quarter-period is $\frac{2\pi}{4} = \frac{\pi}{2}$. The cycle begins at $x = \pi$. Add quarter-periods to generate x-values for the key points.

$x = \pi$

$x = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$

$x = \frac{3\pi}{2} + \frac{\pi}{2} = 2\pi$

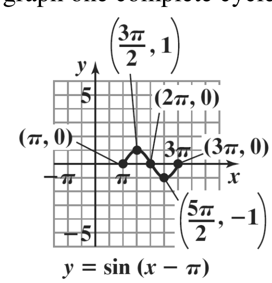
$x = 2\pi + \frac{\pi}{2} = \frac{5\pi}{2}$

$x = \frac{5\pi}{2} + \frac{\pi}{2} = 3\pi$

Evaluate the function at each value of x.

x	$y = \sin(x - \pi)$	coordinates
π	$y = \sin(\pi - \pi)$ $= \sin 0 = 0$	$(\pi, 0)$
$\frac{3\pi}{2}$	$y = \sin\left(\frac{3\pi}{2} - \pi\right)$ $= \sin\frac{\pi}{2} = 1$	$\left(\frac{3\pi}{2}, 1\right)$
2π	$y = \sin(2\pi - \pi)$ $= \sin\pi = 0$	$(2\pi, 0)$
$\frac{5\pi}{2}$	$y = \sin\left(\frac{5\pi}{2} - \pi\right)$ $= \sin\frac{3\pi}{2} = -1$	$\left(\frac{5\pi}{2}, -1\right)$
3π	$y = \sin(3\pi - \pi)$ $= \sin 2\pi = 0$	$(3\pi, 0)$

Connect the five points with a smooth curve and graph one complete cycle of the given function.



18. The equation $y = \sin\left(x - \frac{\pi}{2}\right)$ is of the form $y = A\sin(Bx - C)$ with $A = 1$, $B = 1$, and $C = \frac{\pi}{2}$. The amplitude is $|A| = |1| = 1$. The period is $\frac{2\pi}{B} = \frac{2\pi}{1} = 2\pi$. The phase shift is $\frac{C}{B} = \frac{\frac{\pi}{2}}{1} = \frac{\pi}{2}$. The quarter-period is $\frac{2\pi}{4} = \frac{\pi}{2}$. The cycle begins at $x = \frac{\pi}{2}$. Add quarter-periods to generate x-values for the key points.

$x = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{2} = \pi$

$x = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$

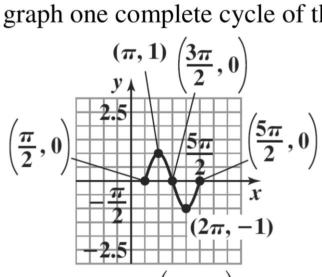
$x = \frac{3\pi}{2} + \frac{\pi}{2} = 2\pi$

$x = 2\pi + \frac{\pi}{2} = \frac{5\pi}{2}$

Evaluate the function at each value of x.

x	$y = \sin\left(x - \frac{\pi}{2}\right)$	coordinates
$\frac{\pi}{2}$	$y = \sin\left(\frac{\pi}{2} - \frac{\pi}{2}\right) = \sin 0 = 0$	$\left(\frac{\pi}{2}, 0\right)$
π	$y = \sin\left(\pi - \frac{\pi}{2}\right) = \sin\frac{\pi}{2} = 1$	$(\pi, 1)$
$\frac{3\pi}{2}$	$y = \sin\left(\frac{3\pi}{2} - \frac{\pi}{2}\right)$ $= \sin \pi = 0$	$\left(\frac{3\pi}{2}, 0\right)$
2π	$y = \sin\left(2\pi - \frac{\pi}{2}\right)$ $= \sin\frac{3\pi}{2} = -1$	$(2\pi, -1)$
$\frac{5\pi}{2}$	$y = \sin\left(\frac{5\pi}{2} - \frac{\pi}{2}\right)$ $= \sin 2\pi = 0$	$\left(\frac{5\pi}{2}, 0\right)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



$y = \sin\left(x - \frac{\pi}{2}\right)$

19. The equation $y = \sin(2x - \pi)$ is of the form $y = A\sin(Bx - C)$ with $A = 1$, $B = 2$, and $C = \pi$. The amplitude is $|A| = |1| = 1$. The period is $\frac{2\pi}{B} = \frac{2\pi}{2} = \pi$. The phase shift is $\frac{C}{B} = \frac{\pi}{2}$. The quarter-period is $\frac{\pi}{4}$. The cycle begins at $x = \frac{\pi}{2}$. Add quarter-periods to generate x-values for the key points.

$x = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$

$x = \frac{3\pi}{4} + \frac{\pi}{4} = \pi$

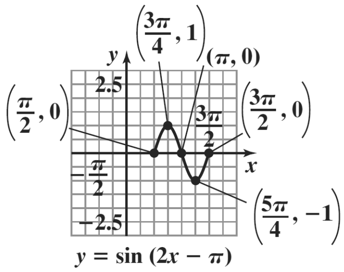
$x = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$

$x = \frac{5\pi}{4} + \frac{\pi}{4} = \frac{3\pi}{2}$

Evaluate the function at each value of x.

x	$y = \sin(2x - \pi)$	coordinates
$\frac{\pi}{2}$	$y = \sin\left(2 \cdot \frac{\pi}{2} - \pi\right)$ $= \sin(\pi - \pi)$ $= \sin 0 = 0$	$\left(\frac{\pi}{2}, 0\right)$
$\frac{3\pi}{4}$	$y = \sin\left(2 \cdot \frac{3\pi}{4} - \pi\right)$ $= \sin\left(\frac{3\pi}{2} - \pi\right)$ $= \sin\frac{\pi}{2} = 1$	$\left(\frac{3\pi}{4}, 1\right)$
π	$y = \sin(2 \cdot \pi - \pi)$ $= \sin(2\pi - \pi)$ $= \sin \pi = 0$	$(\pi, 0)$
$\frac{5\pi}{4}$	$y = \sin\left(2 \cdot \frac{5\pi}{4} - \pi\right)$ $= \sin\left(\frac{5\pi}{2} - \pi\right)$ $= \sin\frac{3\pi}{2} = -1$	$\left(\frac{5\pi}{4}, -1\right)$
$\frac{3\pi}{2}$	$y = \sin\left(2 \cdot \frac{3\pi}{2} - \pi\right)$ $= \sin(3\pi - \pi)$ $= \sin 2\pi = 0$	$\left(\frac{3\pi}{2}, 0\right)$

Connect the five points with a smooth curve and graph one complete cycle of the given function.



20. The equation $y = \sin\left(2x - \frac{\pi}{2}\right)$ is of the form

$y = A\sin(Bx - C)$ with $A = 1$, $B = 2$, and $C = \frac{\pi}{2}$.

The amplitude is $|A| = |1| = 1$.

The period is $\frac{2\pi}{B} = \frac{2\pi}{2} = \pi$.

The phase shift is $\frac{C}{B} = \frac{\frac{\pi}{2}}{2} = \frac{\pi}{2} \cdot \frac{1}{2} = \frac{\pi}{4}$.

The quarter-period is $\frac{\pi}{4}$.

The cycle begins at $x = \frac{\pi}{4}$. Add quarter-periods to generate x-values for the key points.

$x = \frac{\pi}{4}$

$x = \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$

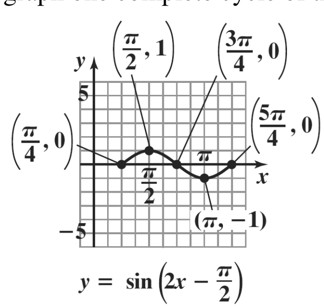
$x = \frac{3\pi}{4} + \frac{\pi}{4} = \pi$

$x = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$

Evaluate the function at each value of x.

x	$y = \sin\left(2x - \frac{\pi}{2}\right)$	coordinates
$\frac{\pi}{4}$	$y = \sin\left(2 \cdot \frac{\pi}{4} - \frac{\pi}{2}\right)$ $= \sin\left(\frac{\pi}{2} - \frac{\pi}{2}\right) = \sin 0 = 0$	$\left(\frac{\pi}{4}, 0\right)$
$\frac{\pi}{2}$	$y = \sin\left(2 \cdot \frac{\pi}{2} - \frac{\pi}{2}\right)$ $= \sin\left(\pi - \frac{\pi}{2}\right) = \sin\frac{\pi}{2} = 1$	$\left(\frac{\pi}{2}, 1\right)$
$\frac{3\pi}{4}$	$y = \sin\left(2 \cdot \frac{3\pi}{4} - \frac{\pi}{2}\right)$ $= \sin\left(\frac{3\pi}{2} - \frac{\pi}{2}\right)$ $= \sin\pi = 0$	$\left(\frac{3\pi}{4}, 0\right)$
π	$y = \sin\left(2 \cdot \pi - \frac{\pi}{2}\right)$ $= \sin\left(2\pi - \frac{\pi}{2}\right)$ $= \sin\frac{3\pi}{2} = -1$	$(\pi, -1)$
$\frac{5\pi}{4}$	$y = \sin\left(2 \cdot \frac{5\pi}{4} - \frac{\pi}{2}\right)$ $= \sin\left(\frac{5\pi}{2} - \frac{\pi}{2}\right)$ $= \sin 2\pi = 0$	$\left(\frac{5\pi}{4}, 0\right)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



21. The equation $y = 3\sin(2x - \pi)$ is of the form $y = A\sin(Bx - C)$ with $A = 3$, $B = 2$, and $C = \pi$. The amplitude is $|A| = |3| = 3$. The period is $\frac{2\pi}{B} = \frac{2\pi}{2} = \pi$. The phase shift is $\frac{C}{B} = \frac{\pi}{2}$. The quarter-period is $\frac{\pi}{4}$. The cycle begins at $x = \frac{\pi}{2}$. Add quarter-periods to generate x-values for the key points.

$x = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$

$x = \frac{3\pi}{4} + \frac{\pi}{4} = \pi$

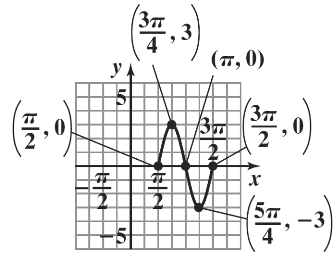
$x = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$

$x = \frac{5\pi}{4} + \frac{\pi}{4} = \frac{3\pi}{2}$

Evaluate the function at each value of x.

x	$y = 3\sin(2x - \pi)$	coordinates
$\frac{\pi}{2}$	$y = 3\sin\left(2 \cdot \frac{\pi}{2} - \pi\right)$ $= 3\sin(\pi - \pi)$ $= 3\sin 0 = 3 \cdot 0 = 0$	$\left(\frac{\pi}{2}, 0\right)$
$\frac{3\pi}{4}$	$y = 3\sin\left(2 \cdot \frac{3\pi}{4} - \pi\right)$ $= 3\sin\left(\frac{3\pi}{2} - \pi\right)$ $= 3\sin\frac{\pi}{2} = 3 \cdot 1 = 3$	$\left(\frac{3\pi}{4}, 3\right)$
π	$y = 3\sin(2 \cdot \pi - \pi)$ $= 3\sin(2\pi - \pi)$ $= 3\sin\pi = 3 \cdot 0 = 0$	$(\pi, 0)$
$\frac{5\pi}{4}$	$y = 3\sin\left(2 \cdot \frac{5\pi}{4} - \pi\right)$ $= 3\sin\left(\frac{5\pi}{2} - \pi\right)$ $= 3\sin\frac{3\pi}{2}$ $= 3(-1) = -3$	$\left(\frac{5\pi}{4}, -3\right)$
$\frac{3\pi}{2}$	$y = 3\sin\left(2 \cdot \frac{3\pi}{2} - \pi\right)$ $= 3\sin(3\pi - \pi)$ $= 3\sin 2\pi = 3 \cdot 0 = 0$	$\left(\frac{3\pi}{2}, 0\right)$

Connect the five points with a smooth curve and graph one complete cycle of the given function.



$y = 3\sin(2x - \pi)$

22. The equation $y = 3\sin\left(2x - \frac{\pi}{2}\right)$ is of the form $y = A\sin(Bx - C)$ with $A = 3$, $B = 2$, and $C = \frac{\pi}{2}$.

The amplitude is $|A| = |3| = 3$.

The period is $\frac{2\pi}{B} = \frac{2\pi}{2} = \pi$.

The phase shift is $\frac{C}{B} = \frac{\frac{\pi}{2}}{2} = \frac{\pi}{2} \cdot \frac{1}{2} = \frac{\pi}{4}$.

The quarter-period is $\frac{\pi}{4}$.

The cycle begins at $x = \frac{\pi}{4}$. Add quarter-periods to generate x-values for the key points.

$x = \frac{\pi}{4}$

$x = \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$

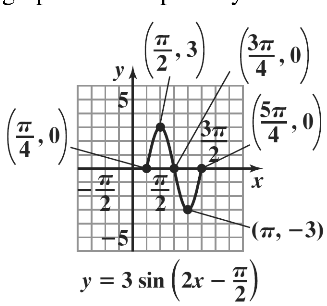
$x = \frac{3\pi}{4} + \frac{\pi}{4} = \pi$

$x = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$

Evaluate the function at each value of x.

x	$y = 3\sin\left(2x - \frac{\pi}{2}\right)$	coordinates
$\frac{\pi}{4}$	$y = 3\sin\left(2\cdot\frac{\pi}{4} - \frac{\pi}{2}\right)$ $= \sin\left(\frac{\pi}{2} - \frac{\pi}{2}\right)$ $= 3\sin 0 = 3\cdot 0 = 0$	$\left(\frac{\pi}{4}, 0\right)$
$\frac{\pi}{2}$	$y = 3\sin\left(2\cdot\frac{\pi}{2} - \frac{\pi}{2}\right)$ $= 3\sin\left(\pi - \frac{\pi}{2}\right)$ $= 3\sin\frac{\pi}{2} = 3\cdot 1 = 3$	$\left(\frac{\pi}{2}, 3\right)$
$\frac{3\pi}{4}$	$y = 3\sin\left(2\cdot\frac{3\pi}{4} - \frac{\pi}{2}\right)$ $= 3\sin\left(\frac{3\pi}{2} - \frac{\pi}{2}\right)$ $= 3\sin\pi = 3\cdot 0 = 0$	$\left(\frac{3\pi}{4}, 0\right)$
π	$y = 3\sin\left(2\cdot\pi - \frac{\pi}{2}\right)$ $= 3\sin\left(2\pi - \frac{\pi}{2}\right)$ $= 3\sin\frac{3\pi}{2} = 3\cdot(-1) = -3$	$(\pi, -3)$
$\frac{5\pi}{4}$	$y = 3\sin\left(2\cdot\frac{5\pi}{4} - \frac{\pi}{2}\right)$ $= 3\sin\left(\frac{5\pi}{2} - \frac{\pi}{2}\right)$ $= 3\sin 2\pi = 3\cdot 0 = 0$	$\left(\frac{5\pi}{4}, 0\right)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



23. $y = \frac{1}{2}\sin\left(x + \frac{\pi}{2}\right) = \frac{1}{2}\sin\left(x - \left(-\frac{\pi}{2}\right)\right)$

The equation $y = \frac{1}{2}\sin\left(x - \left(-\frac{\pi}{2}\right)\right)$ is of the form $y = A\sin(Bx - C)$ with $A = \frac{1}{2}$, $B = 1$, and $C = -\frac{\pi}{2}$. The amplitude is $|A| = \left|\frac{1}{2}\right| = \frac{1}{2}$. The period is $\frac{2\pi}{B} = \frac{2\pi}{1} = 2\pi$. The phase shift is $\frac{C}{B} = \frac{-\frac{\pi}{2}}{1} = -\frac{\pi}{2}$. The quarter-period is $\frac{2\pi}{4} = \frac{\pi}{2}$. The cycle begins at $x = -\frac{\pi}{2}$. Add quarter-periods to generate x-values for the key points.

$x = -\frac{\pi}{2}$

$x = -\frac{\pi}{2} + \frac{\pi}{2} = 0$

$x = 0 + \frac{\pi}{2} = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{2} = \pi$

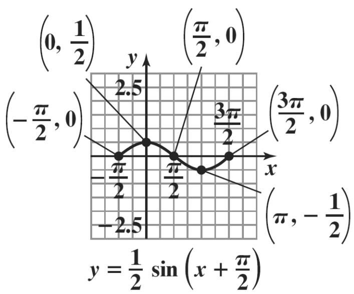
$x = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$

Evaluate the function at each value of x.

x	$y = \frac{1}{2}\sin\left(x + \frac{\pi}{2}\right)$	coordinates
$-\frac{\pi}{2}$	$y = \frac{1}{2}\sin\left(-\frac{\pi}{2} + \frac{\pi}{2}\right)$ $= \frac{1}{2}\sin 0 = \frac{1}{2}\cdot 0 = 0$	$\left(-\frac{\pi}{2}, 0\right)$
0	$y = \frac{1}{2}\sin\left(0 + \frac{\pi}{2}\right)$ $= \frac{1}{2}\sin\frac{\pi}{2} = \frac{1}{2}\cdot 1 = \frac{1}{2}$	$\left(0, \frac{1}{2}\right)$
$\frac{\pi}{2}$	$y = \frac{1}{2}\sin\left(\frac{\pi}{2} + \frac{\pi}{2}\right)$ $= \frac{1}{2}\sin\pi = \frac{1}{2}\cdot 0 = 0$	$\left(\frac{\pi}{2}, 0\right)$

π	$y=\frac{1}{2}\sin\left(\pi+\frac{\pi}{2}\right)$ $=\frac{1}{2}\sin\frac{3\pi}{2}$ $=\frac{1}{2}\cdot(-1)=-\frac{1}{2}$	$\left(\pi,-\frac{1}{2}\right)$
$\frac{3\pi}{2}$	$y=\frac{1}{2}\sin\left(\frac{3\pi}{2}+\frac{\pi}{2}\right)$ $=\frac{1}{2}\sin 2\pi$ $=\frac{1}{2}\cdot 0=0$	$\left(\frac{3\pi}{2},0\right)$

Connect the five points with a smooth curve and graph one complete cycle of the given function.



24. $y=\frac{1}{2}\sin(x+\pi)=\frac{1}{2}\sin(x-(-\pi))$

The equation $y=\frac{1}{2}\sin(x-(-\pi))$ is of the form $y=A\sin(Bx-C)$ with $A=\frac{1}{2}$, $B=1$, and $C=-\pi$.

The amplitude is $|A|=\left|\frac{1}{2}\right|=\frac{1}{2}$. The period is $\frac{2\pi}{B}=\frac{2\pi}{1}=2\pi$. The phase shift is $\frac{C}{B}=\frac{-\pi}{1}=-\pi$.

The quarter-period is $\frac{2\pi}{4}=\frac{\pi}{2}$. The cycle begins at $x=-\pi$. Add quarter-periods to generate *x*-values for the key points.

$x=-\pi$

$x=-\pi+\frac{\pi}{2}=-\frac{\pi}{2}$

$x=-\frac{\pi}{2}+\frac{\pi}{2}=0$

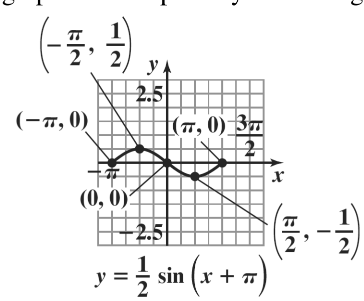
$x=0+\frac{\pi}{2}=\frac{\pi}{2}$

$x=\frac{\pi}{2}+\frac{\pi}{2}=\pi$

Evaluate the function at each value of *x*.

x	$y=\frac{1}{2}\sin(x+\pi)$	coordinates
$-\pi$	$y=\frac{1}{2}\sin(-\pi+\pi)$ $=\frac{1}{2}\sin 0=\frac{1}{2}\cdot 0=0$	$(-\pi, 0)$
$-\frac{\pi}{2}$	$y=\frac{1}{2}\sin\left(-\frac{\pi}{2}+\pi\right)$ $=\frac{1}{2}\sin\frac{\pi}{2}=\frac{1}{2}\cdot 1=\frac{1}{2}$	$\left(-\frac{\pi}{2},\frac{1}{2}\right)$
0	$y=\frac{1}{2}\sin(0+\pi)$ $=\frac{1}{2}\sin\pi=\frac{1}{2}\cdot 0=0$	$(0, 0)$
$\frac{\pi}{2}$	$y=\frac{1}{2}\sin\left(\frac{\pi}{2}+\pi\right)$ $=\frac{1}{2}\sin\frac{3\pi}{2}=\frac{1}{2}\cdot(-1)=-\frac{1}{2}$	$\left(\frac{\pi}{2},-\frac{1}{2}\right)$
π	$y=\frac{1}{2}\sin(\pi+\pi)$ $=\frac{1}{2}\sin 2\pi=\frac{1}{2}\cdot 0=0$	$(\pi, 0)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



25. $y = -2\sin\left(2x + \frac{\pi}{2}\right) = -2\sin\left(2x - \left(-\frac{\pi}{2}\right)\right)$

The equation $y = -2\sin\left(2x - \left(-\frac{\pi}{2}\right)\right)$ is of the form $y = A\sin(Bx - C)$ with $A = -2$, $B = 2$, and $C = -\frac{\pi}{2}$. The amplitude is $|A| = |-2| = 2$. The period is $\frac{2\pi}{B} = \frac{2\pi}{2} = \pi$. The phase shift is $\frac{C}{B} = \frac{-\frac{\pi}{2}}{2} = -\frac{\pi}{2}\cdot\frac{1}{2} = -\frac{\pi}{4}$. The quarter-period is $\frac{\pi}{4}$. The cycle begins at $x = -\frac{\pi}{4}$. Add quarter-periods to generate x-values for the key points.

$x = -\frac{\pi}{4}$

$x = -\frac{\pi}{4} + \frac{\pi}{4} = 0$

$x = 0 + \frac{\pi}{4} = \frac{\pi}{4}$

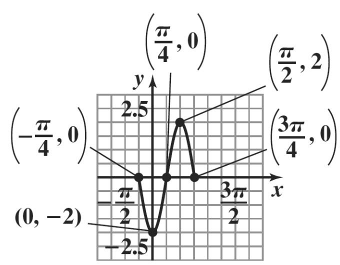
$x = \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$

Evaluate the function at each value of x.

x	$y = -2\sin\left(2x + \frac{\pi}{2}\right)$	coordinates
$-\frac{\pi}{4}$	$y = -2\sin\left(2\cdot\left(-\frac{\pi}{4}\right) + \frac{\pi}{2}\right)$ $= -2\sin\left(-\frac{\pi}{2} + \frac{\pi}{2}\right)$ $= -2\sin 0 = -2\cdot 0 = 0$	$\left(-\frac{\pi}{4}, 0\right)$
0	$y = -2\sin\left(2\cdot 0 + \frac{\pi}{2}\right)$ $= -2\sin\left(0 + \frac{\pi}{2}\right)$ $= -2\sin\frac{\pi}{2}$ $= -2\cdot 1 = -2$	$(0, -2)$
$\frac{\pi}{4}$	$y = -2\sin\left(2\cdot\frac{\pi}{4} + \frac{\pi}{2}\right)$ $= -2\sin\left(\frac{\pi}{2} + \frac{\pi}{2}\right)$ $= -2\sin\pi$ $= -2\cdot 0 = 0$	$\left(\frac{\pi}{4}, 0\right)$
$\frac{\pi}{2}$	$y = -2\sin\left(2\cdot\frac{\pi}{2} + \frac{\pi}{2}\right)$ $= -2\sin\left(\pi + \frac{\pi}{2}\right)$ $= -2\sin\frac{3\pi}{2}$ $= -2(-1) = 2$	$\left(\frac{\pi}{2}, 2\right)$
$\frac{3\pi}{4}$	$y = -2\sin\left(2\cdot\frac{3\pi}{4} + \frac{\pi}{2}\right)$ $= -2\sin\left(\frac{3\pi}{2} + \frac{\pi}{2}\right)$ $= -2\sin 2\pi$ $= -2\cdot 0 = 0$	$\left(\frac{3\pi}{4}, 0\right)$

Connect the five points with a smooth curve and graph one complete cycle of the given function.



$y = -2\sin\left(2x + \frac{\pi}{2}\right)$

26. $y = -3\sin\left(2x + \frac{\pi}{2}\right) = -3\sin\left(2x - \left(-\frac{\pi}{2}\right)\right)$

The equation $y = -3\sin\left(2x - \left(-\frac{\pi}{2}\right)\right)$ is of the form $y = A\sin(Bx - C)$ with $A = -3$, $B = 2$, and $C = -\frac{\pi}{2}$. The amplitude is $|A| = |-3| = 3$. The period is $\frac{2\pi}{B} = \frac{2\pi}{2} = \pi$. The phase shift is $\frac{C}{B} = \frac{-\frac{\pi}{2}}{2} = -\frac{\pi}{2}\cdot\frac{1}{2} = -\frac{\pi}{4}$. The quarter-period is $\frac{\pi}{4}$. The cycle begins at $x = -\frac{\pi}{4}$. Add quarter-periods to generate x-values for the key points.

$x = -\frac{\pi}{4}$

$x = -\frac{\pi}{4} + \frac{\pi}{4} = 0$

$x = 0 + \frac{\pi}{4} = \frac{\pi}{4}$

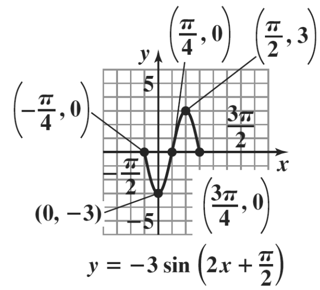
$x = \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$

Evaluate the function at each value of x.

x	$y = -3\sin\left(2x + \frac{\pi}{2}\right)$	coordinates
$-\frac{\pi}{4}$	$y = -3\sin\left(2\cdot\left(-\frac{\pi}{4}\right) + \frac{\pi}{2}\right)$ $= -3\sin\left(-\frac{\pi}{2} + \frac{\pi}{2}\right)$ $= -3\sin 0 = -3\cdot 0 = 0$	$\left(-\frac{\pi}{4}, 0\right)$
0	$y = -3\sin\left(2\cdot 0 + \frac{\pi}{2}\right)$ $= -3\sin\left(0 + \frac{\pi}{2}\right)$ $= -3\sin\frac{\pi}{2} = -3\cdot 1 = -3$	$(0, -3)$
$\frac{\pi}{4}$	$y = -3\sin\left(2\cdot\frac{\pi}{4} + \frac{\pi}{2}\right)$ $= -3\sin\left(\frac{\pi}{2} + \frac{\pi}{2}\right)$ $= -3\sin\pi = -3\cdot 0 = 0$	$\left(\frac{\pi}{4}, 0\right)$
$\frac{\pi}{2}$	$y = -3\sin\left(2\cdot\frac{\pi}{2} + \frac{\pi}{2}\right)$ $= -3\sin\left(\pi + \frac{\pi}{2}\right)$ $= -3\sin\frac{3\pi}{2} = -3\cdot(-1) = 3$	$\left(\frac{\pi}{2}, 3\right)$
$\frac{3\pi}{4}$	$y = -3\sin\left(2\cdot\frac{3\pi}{4} + \frac{\pi}{2}\right)$ $= -3\sin\left(\frac{3\pi}{2} + \frac{\pi}{2}\right)$ $= -3\sin 2\pi = -3\cdot 0 = 0$	$\left(\frac{3\pi}{4}, 0\right)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



27. $y = 3\sin(\pi x + 2)$

The equation $y = 3\sin(\pi x - (-2))$ is of the form $y = A\sin(Bx - C)$ with $A = 3$, $B = \pi$, and $C = -2$. The amplitude is $|A| = |3| = 3$. The period is $\frac{2\pi}{B} = \frac{2\pi}{\pi} = 2$. The phase shift is $\frac{C}{B} = \frac{-2}{\pi} = -\frac{2}{\pi}$. The quarter-period is $\frac{2}{4} = \frac{1}{2}$. The cycle begins at $x = -\frac{2}{\pi}$. Add quarter-periods to generate x-values for the key points.

$x = -\frac{2}{\pi}$

$x = -\frac{2}{\pi} + \frac{1}{2} = \frac{\pi - 4}{2\pi}$

$x = \frac{\pi - 4}{2\pi} + \frac{1}{2} = \frac{\pi - 2}{\pi}$

$x = \frac{\pi - 2}{\pi} + \frac{1}{2} = \frac{3\pi - 4}{2\pi}$

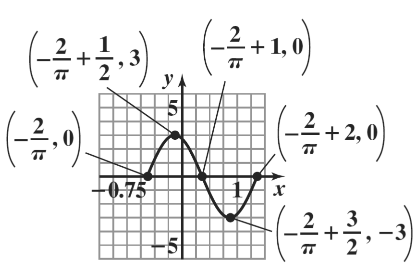
$x = \frac{3\pi - 4}{2\pi} + \frac{1}{2} = \frac{2\pi - 2}{\pi}$

Evaluate the function at each value of x.

x	$y = 3\sin(\pi x + 2)$	coordinates
$-\frac{2}{\pi}$	$y = 3\sin\left(\pi\left(-\frac{2}{\pi}\right) + 2\right)$ $= 3\sin(-2 + 2)$ $= 3\sin 0 = 3\cdot 0 = 0$	$\left(-\frac{2}{\pi}, 0\right)$

$\frac{\pi-4}{2\pi}$	$y=3\sin\left(\pi\left(\frac{\pi-4}{2\pi}\right)+2\right)$ $=3\sin\left(\frac{\pi-4}{2}+2\right)$ $=3\sin\left(\frac{\pi}{2}-2+2\right)$ $=3\sin\frac{\pi}{2}$ $=3\cdot 1=3$	$\left(\frac{\pi-4}{2\pi}, 3\right)$
$\frac{\pi-2}{\pi}$	$y=3\sin\left(\pi\left(\frac{\pi-2}{\pi}\right)+2\right)$ $=3\sin(\pi-2+2)$ $=3\sin\pi=3\cdot 0=0$	$\left(\frac{\pi-2}{\pi}, 0\right)$
$\frac{3\pi-4}{2\pi}$	$y=3\sin\left(\pi\left(\frac{3\pi-4}{2\pi}\right)+2\right)$ $=3\sin\left(\frac{3\pi-4}{2}+2\right)$ $=3\sin\left(\frac{3\pi}{2}-2+2\right)$ $=3\sin\frac{3\pi}{2}$ $=3(-1)=-3$	$\left(\frac{5\pi}{4}, -3\right)$
$\frac{2\pi-2}{\pi}$	$y=3\sin\left(\pi\left(\frac{2\pi-2}{\pi}\right)+2\right)$ $=3\sin(2\pi-2+2)$ $=3\sin 2\pi=3\cdot 0=0$	$\left(\frac{2\pi-2}{\pi}, 0\right)$

Connect the five points with a smooth curve and graph one complete cycle of the given function.



$y = 3\sin(\pi x + 2)$

28. $y=3\sin(2\pi x+4)=3\sin(2\pi x-(-4))$

The equation $y=3\sin(2\pi x-(-4))$ is of the form $y=A\sin(Bx-C)$ with $A = 3$, $B=2\pi$, and $C = -4$. The amplitude is $|A|=|3|=3$. The period is $\frac{2\pi}{B}=\frac{2\pi}{2\pi}=1$. The phase shift is $\frac{C}{B}=\frac{-4}{2\pi}=-\frac{2}{\pi}$. The quarter-period is $\frac{1}{4}$. The cycle begins at $x=-\frac{2}{\pi}$. Add quarter-periods to generate x-values for the key points.

$x=-\frac{2}{\pi}$

$x=-\frac{2}{\pi}+\frac{1}{4}=\frac{\pi-8}{4\pi}$

$x=\frac{\pi-8}{4\pi}+\frac{1}{4}=\frac{\pi-4}{2\pi}$

$x=\frac{\pi-4}{2\pi}+\frac{1}{4}=\frac{3\pi-8}{4\pi}$

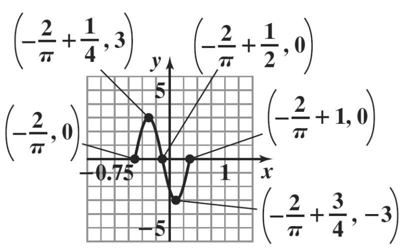
$x=\frac{3\pi-8}{4\pi}+\frac{1}{4}=\frac{\pi-2}{\pi}$

Evaluate the function at each value of x.

x	$y=3\sin(2\pi x+4)$	coordinates
$-\frac{2}{\pi}$	$y=3\sin\left(2\pi\left(-\frac{2}{\pi}\right)+4\right)$ $=3\sin(-4+4)$ $=3\sin 0=3\cdot 0=0$	$\left(-\frac{2}{\pi}, 0\right)$
$\frac{\pi-8}{4\pi}$	$y=3\sin\left(2\pi\left(\frac{\pi-8}{4\pi}\right)+4\right)$ $=3\sin\left(\frac{\pi-8}{2}+4\right)$ $=3\sin\left(\frac{\pi}{2}-4+4\right)$ $=3\sin\frac{\pi}{2}=3\cdot 1=3$	$\left(\frac{\pi-8}{4\pi}, 3\right)$
$\frac{\pi-4}{2\pi}$	$y=3\sin\left(2\pi\left(\frac{\pi-4}{2\pi}\right)+4\right)$ $=3\sin(\pi-4+4)$ $=3\sin\pi=3\cdot 0=0$	$\left(\frac{\pi-4}{2\pi}, 0\right)$

$\frac{3\pi-8}{4\pi}$	$y=3\sin\left(2\pi\left(\frac{3\pi-8}{4\pi}\right)+4\right)$ $=3\sin\left(\frac{3\pi-8}{2}+4\right)$ $=3\sin\left(\frac{3\pi}{2}-4+4\right)$ $=3\sin\frac{3\pi}{2}=3(-1)=-3$	$\left(\frac{3\pi-8}{4\pi},-3\right)$
$\frac{\pi-2}{\pi}$	$y=3\sin\left(2\pi\left(\frac{\pi-2}{\pi}\right)+4\right)$ $=3\sin(2\pi-4+4)$ $=3\sin 2\pi=3\cdot 0=0$	$\left(\frac{\pi-2}{\pi},0\right)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



$y = 3\sin(2\pi x + 4)$

29. $y=-2\sin(2\pi x+4\pi)=-2\sin(2\pi x-(-4\pi))$

The equation $y=-2\sin(2\pi x-(-4\pi))$ is of the form $y=A\sin(Bx-C)$ with $A=-2$, $B=2\pi$, and $C=-4\pi$. The amplitude is $|A|=|-2|=2$. The period is $\frac{2\pi}{B}=\frac{2\pi}{2\pi}=1$. The phase shift is $\frac{C}{B}=\frac{-4\pi}{2\pi}=-2$. The quarter-period is $\frac{1}{4}$. The cycle begins at $x=-2$. Add quarter-periods to generate x-values for the key points.

$x=-2$

$x=-2+\frac{1}{4}=-\frac{7}{4}$

$x=-\frac{7}{4}+\frac{1}{4}=-\frac{3}{2}$

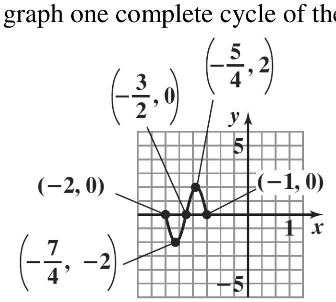
$x=-\frac{3}{2}+\frac{1}{4}=-\frac{5}{4}$

$x=-\frac{5}{4}+\frac{1}{4}=-1$

Evaluate the function at each value of x.

x	$y=-2\sin(2\pi x+4\pi)$	coordinates
-2	$y=-2\sin(2\pi(-2)+4\pi)$ $=-2\sin(-4\pi+4\pi)$ $=-2\sin 0$ $=-2\cdot 0=0$	$(-2, 0)$
$-\frac{7}{4}$	$y=-2\sin\left(2\pi\left(-\frac{7}{4}\right)+4\pi\right)$ $=-2\sin\left(-\frac{7\pi}{2}+4\pi\right)$ $=-2\sin\frac{\pi}{2}=-2\cdot 1=-2$	$\left(-\frac{7}{4},-2\right)$
$-\frac{3}{2}$	$y=-2\sin\left(2\pi\left(-\frac{3}{2}\right)+4\pi\right)$ $=-2\sin(-3\pi+4\pi)$ $=-2\sin\pi=-2\cdot 0=0$	$\left(-\frac{3}{2},0\right)$
$-\frac{5}{4}$	$y=-2\sin\left(2\pi\left(-\frac{5}{4}\right)+4\pi\right)$ $=-2\sin\left(-\frac{5\pi}{2}+4\pi\right)$ $=-2\sin\frac{3\pi}{2}$ $=-2(-1)=2$	$\left(-\frac{5}{4},2\right)$
-1	$y=-2\sin(2\pi(-1)+4\pi)$ $=-2\sin(-2\pi+4\pi)$ $=-2\sin 2\pi$ $=-2\cdot 0=0$	$(-1, 0)$

Connect the five points with a smooth curve and graph one complete cycle of the given function.



$y = -2\sin(2\pi x + 4\pi)$

30. $y = -3\sin(2\pi x + 4\pi) = -3\sin(2\pi x - (-4\pi))$

The equation $y = -3\sin(2\pi x - (-4\pi))$ is of the form $y = A\sin(Bx - C)$ with $A = -3$, $B = 2\pi$, and $C = -4\pi$. The amplitude is $|A| = |-3| = 3$. The period is $\frac{2\pi}{B} = \frac{2\pi}{2\pi} = 1$. The phase shift is $\frac{C}{B} = \frac{-4\pi}{2\pi} = -2$. The quarter-period is $\frac{1}{4}$. The cycle begins at $x = -2$. Add quarter-periods to generate x-values for the key points.

$x = -2$

$x = -2 + \frac{1}{4} = -\frac{7}{4}$

$x = -\frac{7}{4} + \frac{1}{4} = -\frac{3}{2}$

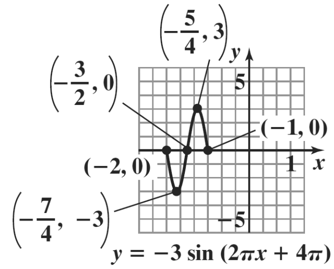
$x = -\frac{3}{2} + \frac{1}{4} = -\frac{5}{4}$

$x = -\frac{5}{4} + \frac{1}{4} = -1$

Evaluate the function at each value of x.

x	$y = -3\sin(2\pi x + 4\pi)$	coordinates
-2	$y = -3\sin(2\pi(-2) + 4\pi)$ $= -3\sin(-4\pi + 4\pi)$ $= -3\sin 0 = -3 \cdot 0 = 0$	$(-2, 0)$
$-\frac{7}{4}$	$y = -3\sin\left(2\pi\left(-\frac{7}{4}\right) + 4\pi\right)$ $= -3\sin\left(-\frac{7\pi}{2} + 4\pi\right)$ $= -3\sin\frac{\pi}{2} = -3 \cdot 1 = -3$	$\left(-\frac{7}{4}, -3\right)$
$-\frac{3}{2}$	$y = -3\sin\left(2\pi\left(-\frac{3}{2}\right) + 4\pi\right)$ $= -3\sin(-3\pi + 4\pi)$ $= -3\sin\pi = -3 \cdot 0 = 0$	$\left(-\frac{3}{2}, 0\right)$
$-\frac{5}{4}$	$y = -3\sin\left(2\pi\left(-\frac{5}{4}\right) + 4\pi\right)$ $= -3\sin\left(-\frac{5\pi}{2} + 4\pi\right)$ $= -3\sin\frac{3\pi}{2} = -3(-1) = 3$	$\left(-\frac{5}{4}, 3\right)$
-1	$y = -3\sin(2\pi(-1) + 4\pi)$ $= -3\sin(-2\pi + 4\pi)$ $= -3\sin 2\pi = -3 \cdot 0 = 0$	$(-1, 0)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



31. The equation $y = 2\cos x$ is of the form $y = A\cos x$ with $A = 2$. Thus, the amplitude is $|A| = |2| = 2$. The period is 2π. The quarter-period is $\frac{2\pi}{4}$ or $\frac{\pi}{2}$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{\pi}{2} = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{2} = \pi$

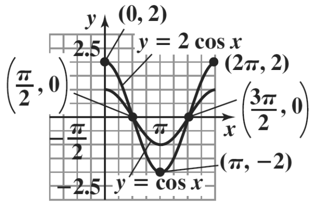
$x = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$

$x = \frac{3\pi}{2} + \frac{\pi}{2} = 2\pi$

Evaluate the function at each value of x.

x	$y = 2\cos x$	coordinates
0	$y = 2\cos 0$ $= 2 \cdot 1 = 2$	$(0, 2)$
$\frac{\pi}{2}$	$y = 2\cos\frac{\pi}{2}$ $= 2 \cdot 0 = 0$	$\left(\frac{\pi}{2}, 0\right)$
π	$y = 2\cos\pi$ $= 2 \cdot (-1) = -2$	$(\pi, -2)$
$\frac{3\pi}{2}$	$y = 2\cos\frac{3\pi}{2}$ $= 2 \cdot 0 = 0$	$\left(\frac{3\pi}{2}, 0\right)$
2π	$y = 2\cos 2\pi$ $= 2 \cdot 1 = 2$	$(2\pi, 2)$

Connect the five points with a smooth curve and graph one complete cycle of the given function with the graph of $y = 2\cos x$.



32. The equation $y = 3\cos x$ is of the form $y = A\cos x$ with $A = 3$. Thus, the amplitude is $|A| = |3| = 3$. The period is 2π. The quarter-period is $\frac{2\pi}{4}$ or $\frac{\pi}{2}$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{\pi}{2} = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{2} = \pi$

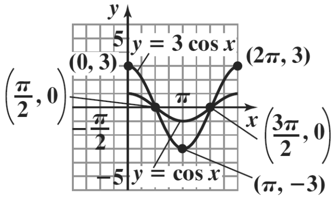
$x = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$

$x = \frac{3\pi}{2} + \frac{\pi}{2} = 2\pi$

Evaluate the function at each value of x.

x	$y = 3\cos x$	coordinates
0	$y = 3\cos 0 = 3\cdot 1 = 3$	(0, 3)
$\frac{\pi}{2}$	$y = 3\cos\frac{\pi}{2} = 3\cdot 0 = 0$	$\left(\frac{\pi}{2}, 0\right)$
π	$y = 3\cos\pi = 3\cdot(-1) = -3$	$(\pi, -3)$
$\frac{3\pi}{2}$	$y = 3\cos\frac{3\pi}{2} = 3\cdot 0 = 0$	$\left(\frac{3\pi}{2}, 0\right)$
2π	$y = 3\cos 2\pi = 3\cdot 1 = 3$	$(2\pi, 3)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function with the graph of $y = \cos x$.



33. The equation $y = -2\cos x$ is of the form $y = A\cos x$ with $A = -2$. Thus, the amplitude is $|A| = |-2| = 2$. The period is 2π. The quarter-period is $\frac{2\pi}{4}$ or $\frac{\pi}{2}$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{\pi}{2} = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{2} = \pi$

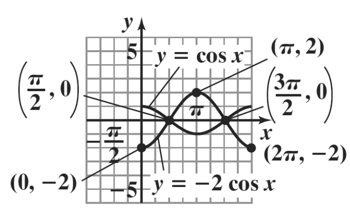
$x = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$

$x = \frac{3\pi}{2} + \frac{\pi}{2} = 2\pi$

Evaluate the function at each value of x.

x	$y = -2\cos x$	coordinates
0	$y = -2\cos 0$ $= -2\cdot 1 = -2$	(0, –2)
$\frac{\pi}{2}$	$y = -2\cos\frac{\pi}{2}$ $= -2\cdot 0 = 0$	$\left(\frac{\pi}{2}, 0\right)$
π	$y = -2\cos\pi$ $= -2\cdot(-1) = 2$	$(\pi, 2)$
$\frac{3\pi}{2}$	$y = -2\cos\frac{3\pi}{2}$ $= -2\cdot 0 = 0$	$\left(\frac{3\pi}{2}, 0\right)$
2π	$y = -2\cos 2\pi$ $= -2\cdot 1 = -2$	$(2\pi, -2)$

Connect the five points with a smooth curve and graph one complete cycle of the given function with the graph of $y = \cos x$.



34. The equation $y = -3\cos x$ is of the form $y = A\cos x$ with $A = -3$. Thus, the amplitude is $|A| = |-3| = 3$. The period is 2π. The quarter-period is $\frac{2\pi}{4}$ or $\frac{\pi}{2}$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{\pi}{2} = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{2} = \pi$

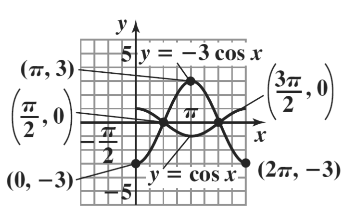
$x = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$

$x = \frac{3\pi}{2} + \frac{\pi}{2} = 2\pi$

Evaluate the function at each value of x.

x	$y = -3\cos x$	coordinates
0	$y = -3\cos 0 = -3 \cdot 1 = -3$	(0, –3)
$\frac{\pi}{2}$	$y = -3\cos\frac{\pi}{2} = -3 \cdot 0 = 0$	$\left(\frac{\pi}{2}, 0\right)$
π	$y = -3\cos\pi = -3 \cdot (-1) = 3$	$(\pi, 3)$
$\frac{3\pi}{2}$	$y = -3\cos\frac{3\pi}{2} = -3 \cdot 0 = 0$	$\left(\frac{3\pi}{2}, 0\right)$
2π	$y = -3\cos 2\pi = -3 \cdot 1 = -3$	$(2\pi, -3)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function with the graph of $y = \cos x$.



35. The equation $y = \cos 2x$ is of the form $y = A\cos Bx$ with $A = 1$ and $B = 2$. Thus, the amplitude is $|A| = |1| = 1$. The period is $\frac{2\pi}{B} = \frac{2\pi}{2} = \pi$. The quarter-period is $\frac{\pi}{4}$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{\pi}{4} = \frac{\pi}{4}$

$x = \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$

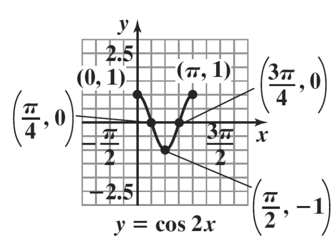
$x = \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$

$x = \frac{3\pi}{4} + \frac{\pi}{4} = \pi$

Evaluate the function at each value of x.

x	$y = \cos 2x$	coordinates
0	$y = \cos(2 \cdot 0)$ $= \cos 0 = 1$	(0, 1)
$\frac{\pi}{4}$	$y = \cos\left(2 \cdot \frac{\pi}{4}\right)$ $= \cos\frac{\pi}{2} = 0$	$\left(\frac{\pi}{4}, 0\right)$
$\frac{\pi}{2}$	$y = \cos\left(2 \cdot \frac{\pi}{2}\right)$ $= \cos\pi = -1$	$\left(\frac{\pi}{2}, -1\right)$
$\frac{3\pi}{4}$	$y = \cos\left(2 \cdot \frac{3\pi}{4}\right)$ $= \cos\frac{3\pi}{2} = 0$	$\left(\frac{3\pi}{4}, 0\right)$
π	$y = \cos(2 \cdot \pi)$ $= \cos 2\pi = 1$	$(\pi, 1)$

Connect the five points with a smooth curve and graph one complete cycle of the given function.



36. The equation $y = \cos 4x$ is of the form $y = A\cos Bx$ with $A = 1$ and $B = 4$. Thus, the amplitude is $|A| = |1| = 1$. The period is $\frac{2\pi}{B} = \frac{2\pi}{4} = \frac{\pi}{2}$. The quarter-period is $\frac{\frac{\pi}{2}}{4} = \frac{\pi}{2}\cdot\frac{1}{4} = \frac{\pi}{8}$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{\pi}{8} = \frac{\pi}{8}$

$x = \frac{\pi}{8} + \frac{\pi}{8} = \frac{\pi}{4}$

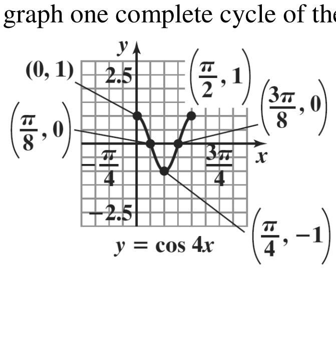
$x = \frac{\pi}{4} + \frac{\pi}{8} = \frac{3\pi}{8}$

$x = \frac{3\pi}{8} + \frac{\pi}{8} = \frac{\pi}{2}$

Evaluate the function at each value of x.

x	$y = \cos 4x$	coordinates
0	$y = \cos(4\cdot 0) = \cos 0 = 1$	(0, 1)
$\frac{\pi}{8}$	$y = \cos\left(4\cdot\frac{\pi}{8}\right) = \cos\frac{\pi}{2} = 0$	$\left(\frac{\pi}{8}, 0\right)$
$\frac{\pi}{4}$	$y = \cos\left(4\cdot\frac{\pi}{4}\right) = \cos\pi = -1$	$\left(\frac{\pi}{4}, -1\right)$
$\frac{3\pi}{8}$	$y = \cos\left(4\cdot\frac{3\pi}{8}\right)$ $= \cos\frac{3\pi}{2} = 0$	$\left(\frac{3\pi}{8}, 0\right)$
$\frac{\pi}{2}$	$y = \cos\left(4\cdot\frac{\pi}{2}\right) = \cos 2\pi = 1$	$\left(\frac{\pi}{2}, 1\right)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



37. The equation $y = 4\cos 2\pi x$ is of the form $y = A\cos Bx$ with $A = 4$ and $B = 2\pi$. Thus, the amplitude is $|A| = |4| = 4$. The period is $\frac{2\pi}{B} = \frac{2\pi}{2\pi} = 1$. The quarter-period is $\frac{1}{4}$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{1}{4} = \frac{1}{4}$

$x = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

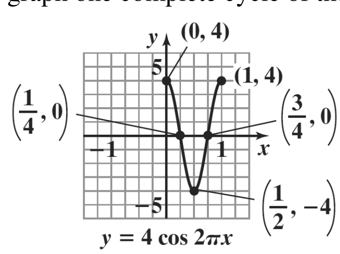
$x = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$

$x = \frac{3}{4} + \frac{1}{4} = 1$

Evaluate the function at each value of x.

x	$y = 4\cos 2\pi x$	coordinates
0	$y = 4\cos(2\pi\cdot 0)$ $= 4\cos 0$ $= 4\cdot 1 = 4$	(0, 4)
$\frac{1}{4}$	$y = 4\cos\left(2\pi\cdot\frac{1}{4}\right)$ $= 4\cos\frac{\pi}{2}$ $= 4\cdot 0 = 0$	$\left(\frac{1}{4}, 0\right)$
$\frac{1}{2}$	$y = 4\cos\left(2\pi\cdot\frac{1}{2}\right)$ $= 4\cos\pi$ $= 4\cdot(-1) = -4$	$\left(\frac{1}{2}, -4\right)$
$\frac{3}{4}$	$y = 4\cos\left(2\pi\cdot\frac{3}{4}\right)$ $= 4\cos\frac{3\pi}{2}$ $= 4\cdot 0 = 0$	$\left(\frac{3}{4}, 0\right)$
1	$y = 4\cos(2\pi\cdot 1)$ $= 4\cos 2\pi$ $= 4\cdot 1 = 4$	(1, 4)

Connect the five points with a smooth curve and graph one complete cycle of the given function.



38. The equation $y = 5\cos 2\pi x$ is of the form $y = A\cos Bx$ with $A = 5$ and $B = 2\pi$. Thus, the amplitude is $|A| = |5| = 5$. The period is $\frac{2\pi}{B} = \frac{2\pi}{2\pi} = 1$. The quarter-period is $\frac{1}{4}$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{1}{4} = \frac{1}{4}$

$x = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

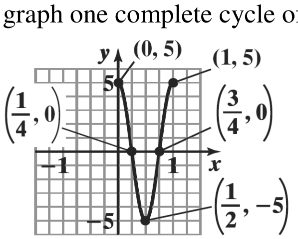
$x = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$

$x = \frac{3}{4} + \frac{1}{4} = 1$

Evaluate the function at each value of x.

x	$y = 5\cos 2\pi x$	coordinates
0	$y = 5\cos(2\pi \cdot 0)$ $= 5\cos 0 = 5 \cdot 1 = 5$	$(0, 5)$
$\frac{1}{4}$	$y = 5\cos\left(2\pi \cdot \frac{1}{4}\right)$ $= 5\cos\frac{\pi}{2} = 5 \cdot 0 = 0$	$\left(\frac{1}{4}, 0\right)$
$\frac{1}{2}$	$y = 5\cos\left(2\pi \cdot \frac{1}{2}\right)$ $= 5\cos\pi = 5 \cdot (-1) = -5$	$\left(\frac{1}{2}, -5\right)$
$\frac{3}{4}$	$y = 5\cos\left(2\pi \cdot \frac{3\pi}{4}\right)$ $= 5\cos\frac{3\pi}{2} = 5 \cdot 0 = 0$	$\left(\frac{3}{4}, 0\right)$
1	$y = 5\cos(2\pi \cdot 1)$ $= 5\cos 2\pi = 5 \cdot 1 = 5$	$(1, 5)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



$y = 5\cos 2\pi x$

39. The equation $y = -4\cos\frac{1}{2}x$ is of the form $y = A\cos Bx$ with $A = -4$ and $B = \frac{1}{2}$. Thus, the amplitude is $|A| = |-4| = 4$. The period is $\frac{2\pi}{B} = \frac{2\pi}{\frac{1}{2}} = 2\pi \cdot 2 = 4\pi$. The quarter-period is $\frac{4\pi}{4} = \pi$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \pi = \pi$

$x = \pi + \pi = 2\pi$

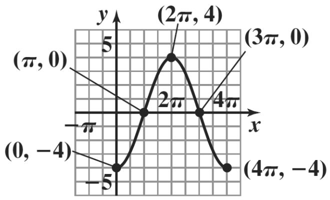
$x = 2\pi + \pi = 3\pi$

$x = 3\pi + \pi = 4\pi$

Evaluate the function at each value of x.

x	$y = -4\cos\frac{1}{2}x$	coordinates
0	$y = -4\cos\left(\frac{1}{2} \cdot 0\right)$ $= -4\cos 0$ $= -4 \cdot 1 = -4$	$(0, -4)$
π	$y = -4\cos\left(\frac{1}{2} \cdot \pi\right)$ $= -4\cos\frac{\pi}{2}$ $= -4 \cdot 0 = 0$	$(\pi, 0)$
2π	$y = -4\cos\left(\frac{1}{2} \cdot 2\pi\right)$ $= -4\cos\pi$ $= -4 \cdot (-1) = 4$	$(2\pi, 4)$
3π	$y = -4\cos\left(\frac{1}{2} \cdot 3\pi\right)$ $= -4\cos\frac{3\pi}{2}$ $= -4 \cdot 0 = 0$	$(3\pi, 0)$
4π	$y = -4\cos\left(\frac{1}{2} \cdot 4\pi\right)$ $= -4\cos 2\pi$ $= -4 \cdot 1 = -4$	$(4\pi, -4)$

Connect the five points with a smooth curve and graph one complete cycle of the given function.



$y = -4\cos\frac{1}{2}x$

40. The equation $y = -3\cos\frac{1}{3}x$ is of the form $y = A\cos Bx$ with $A = -3$ and $B = \frac{1}{3}$. Thus, the amplitude is $|A| = |-3| = 3$. The period is $\frac{2\pi}{B} = \frac{2\pi}{\frac{1}{3}} = 2\pi \cdot 3 = 6\pi$. The quarter-period is $\frac{6\pi}{4} = \frac{3\pi}{2}$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{3\pi}{2} = \frac{3\pi}{2}$

$x = \frac{3\pi}{2} + \frac{3\pi}{2} = 3\pi$

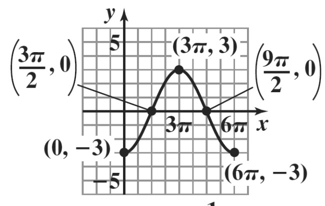
$x = 3\pi + \frac{3\pi}{2} = \frac{9\pi}{2}$

$x = \frac{9\pi}{2} + \frac{3\pi}{2} = 6\pi$

Evaluate the function at each value of x.

x	$y = -3\cos\frac{1}{3}x$	coordinates
0	$y = -3\cos\left(\frac{1}{3}\cdot 0\right)$ $= -3\cos 0 = -3\cdot 1 = -3$	$(0, -3)$
$\frac{3\pi}{2}$	$y = -3\cos\left(\frac{1}{3}\cdot\frac{3\pi}{2}\right)$ $= -3\cos\frac{\pi}{2} = -3\cdot 0 = 0$	$\left(\frac{3\pi}{2}, 0\right)$
3π	$y = -3\cos\left(\frac{1}{3}\cdot 3\pi\right)$ $= -3\cos\pi = -3\cdot(-1) = 3$	$(3\pi, 3)$
$\frac{9\pi}{2}$	$y = -3\cos\left(\frac{1}{3}\cdot\frac{9\pi}{2}\right)$ $= -3\cos\frac{3\pi}{2} = -3\cdot 0 = 0$	$\left(\frac{9\pi}{2}, 0\right)$
6π	$y = -3\cos\left(\frac{1}{3}\cdot 6\pi\right)$ $= -3\cos 2\pi = -3\cdot 1 = -3$	$(6\pi, -3)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



$y = -3\cos\frac{1}{3}x$

41. The equation $y = -\frac{1}{2}\cos\frac{\pi}{3}x$ is of the form $y = A\cos Bx$ with $A = -\frac{1}{2}$ and $B = \frac{\pi}{3}$. Thus, the amplitude is $|A| = \left|-\frac{1}{2}\right| = \frac{1}{2}$. The period is $\frac{2\pi}{B} = \frac{2\pi}{\frac{\pi}{3}} = 2\pi\cdot\frac{3}{\pi} = 6$. The quarter-period is $\frac{6}{4} = \frac{3}{2}$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{3}{2} = \frac{3}{2}$

$x = \frac{3}{2} + \frac{3}{2} = 3$

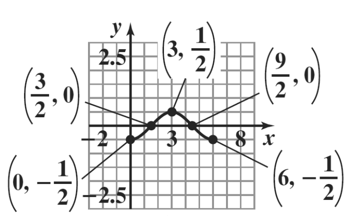
$x = 3 + \frac{3}{2} = \frac{9}{2}$

$x = \frac{9}{2} + \frac{3}{2} = 6$

Evaluate the function at each value of x.

x	$y=-\frac{1}{2}\cos\frac{\pi}{3}x$	coordinates
0	$y=-\frac{1}{2}\cos\left(\frac{\pi}{3}\cdot 0\right)$ $=-\frac{1}{2}\cos 0$ $=-\frac{1}{2}\cdot 1=-\frac{1}{2}$	$\left(0,-\frac{1}{2}\right)$
$\frac{3}{2}$	$y=-\frac{1}{2}\cos\left(\frac{\pi}{3}\cdot\frac{3}{2}\right)$ $=-\frac{1}{2}\cos\frac{\pi}{2}$ $=-\frac{1}{2}\cdot 0=0$	$\left(\frac{3}{2},0\right)$
3	$y=-\frac{1}{2}\cos\left(\frac{\pi}{3}\cdot 3\right)$ $=-\frac{1}{2}\cos\pi$ $=-\frac{1}{2}\cdot(-1)=\frac{1}{2}$	$\left(3,\frac{1}{2}\right)$
$\frac{9}{2}$	$y=-\frac{1}{2}\cos\left(\frac{\pi}{3}\cdot\frac{9}{2}\right)$ $=-\frac{1}{2}\cos\frac{3\pi}{2}$ $=-\frac{1}{2}\cdot 0=0$	$\left(\frac{9}{2},0\right)$
6	$y=-\frac{1}{2}\cos\left(\frac{\pi}{3}\cdot 6\right)$ $=-\frac{1}{2}\cos 2\pi$ $=-\frac{1}{2}\cdot 1=-\frac{1}{2}$	$\left(6,-\frac{1}{2}\right)$

Connect the five points with a smooth curve and graph one complete cycle of the given function.



$y=-\frac{1}{2}\cos\frac{\pi}{3}x$

42. The equation $y=-\frac{1}{2}\cos\frac{\pi}{4}x$ is of the form $y=A\cos Bx$ with $A=-\frac{1}{2}$ and $B=\frac{\pi}{4}$. Thus, the amplitude is $|A|=\left|-\frac{1}{2}\right|=\frac{1}{2}$. The period is $\frac{2\pi}{B}=\frac{2\pi}{\frac{\pi}{4}}=2\pi\cdot\frac{4}{\pi}=8$. The quarter-period is $\frac{8}{4}=2$.

The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x=0$

$x=0+2=2$

$x=2+2=4$

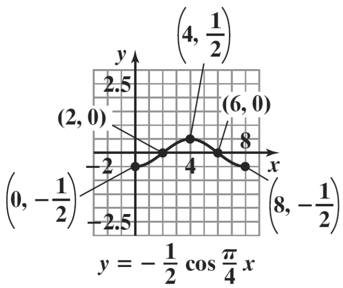
$x=4+2=6$

$x=6+2=8$

Evaluate the function at each value of x.

x	$y=-\frac{1}{2}\cos\frac{\pi}{4}x$	coordinates
0	$y=-\frac{1}{2}\cos\left(\frac{\pi}{4}\cdot 0\right)$ $=-\frac{1}{2}\cos 0=-\frac{1}{2}\cdot 1=-\frac{1}{2}$	$\left(0,-\frac{1}{2}\right)$
2	$y=-\frac{1}{2}\cos\left(\frac{\pi}{4}\cdot 2\right)$ $=-\frac{1}{2}\cos\frac{\pi}{2}=-\frac{1}{2}\cdot 0=0$	(2, 0)
4	$y=-\frac{1}{2}\cos\left(\frac{\pi}{4}\cdot 4\right)$ $=-\frac{1}{2}\cos\pi=-\frac{1}{2}\cdot(-1)=\frac{1}{2}$	$\left(4,\frac{1}{2}\right)$
6	$y=-\frac{1}{2}\cos\left(\frac{\pi}{4}\cdot 6\right)$ $=-\frac{1}{2}\cos\left(\frac{3\pi}{2}\right)=-\frac{1}{2}\cdot 0=0$	(6, 0)
8	$y=-\frac{1}{2}\cos\left(\frac{\pi}{4}\cdot 8\right)$ $=-\frac{1}{2}\cos 2\pi=-\frac{1}{2}\cdot 1=-\frac{1}{2}$	$\left(8,-\frac{1}{2}\right)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



43. The equation $y = \cos\left(x - \frac{\pi}{2}\right)$ is of the form $y = A\cos(Bx - C)$ with $A = 1$, and $B = 1$, and $C = \frac{\pi}{2}$. Thus, the amplitude is $|A| = |1| = 1$. The period is $\frac{2\pi}{B} = \frac{2\pi}{1} = 2\pi$. The phase shift is $\frac{C}{B} = \frac{\frac{\pi}{2}}{1} = \frac{\pi}{2}$. The quarter-period is $\frac{2\pi}{4} = \frac{\pi}{2}$. The cycle begins at $x = \frac{\pi}{2}$. Add quarter-periods to generate x-values for the key points.

$x = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{2} = \pi$

$x = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$

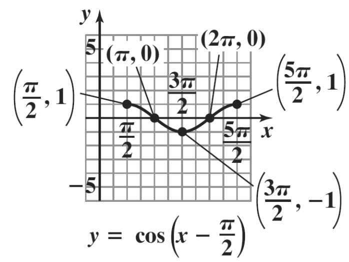
$x = \frac{3\pi}{2} + \frac{\pi}{2} = 2\pi$

$x = 2\pi + \frac{\pi}{2} = \frac{5\pi}{2}$

Evaluate the function at each value of x.

x	coordinates
$\frac{\pi}{2}$	$\left(\frac{\pi}{2}, 1\right)$
π	$(\pi, 0)$
$\frac{3\pi}{2}$	$\left(\frac{3\pi}{2}, -1\right)$
2π	$(2\pi, 0)$
$\frac{5\pi}{2}$	$\left(\frac{5\pi}{2}, 1\right)$

Connect the five points with a smooth curve and graph one complete cycle of the given function



44. The equation $y = \cos\left(x + \frac{\pi}{2}\right)$ is of the form $y = A\cos(Bx - C)$ with $A = 1$, and $B = 1$, and $C = -\frac{\pi}{2}$. Thus, the amplitude is $|A| = |1| = 1$. The period is $\frac{2\pi}{B} = \frac{2\pi}{1} = 2\pi$. The phase shift is $\frac{C}{B} = \frac{-\frac{\pi}{2}}{1} = -\frac{\pi}{2}$. The quarter-period is $\frac{2\pi}{4} = \frac{\pi}{2}$. The cycle begins at $x = -\frac{\pi}{2}$. Add quarter-periods to generate x-values for the key points.

$x = -\frac{\pi}{2}$

$x = -\frac{\pi}{2} + \frac{\pi}{2} = 0$

$x = 0 + \frac{\pi}{2} = \frac{\pi}{2}$

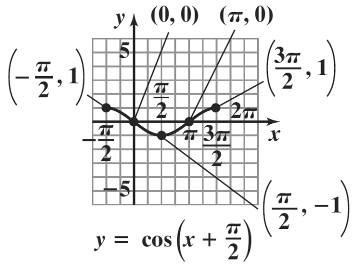
$x = \frac{\pi}{2} + \frac{\pi}{2} = \pi$

$x = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$

Evaluate the function at each value of x.

x	coordinates
$-\frac{\pi}{2}$	$\left(-\frac{\pi}{2}, 1\right)$
0	$(0, 0)$
$\frac{\pi}{2}$	$\left(\frac{\pi}{2}, -1\right)$
π	$(\pi, 0)$
$\frac{3\pi}{2}$	$\left(\frac{3\pi}{2}, 1\right)$

Connect the five points with a smooth curve and graph one complete cycle of the given function



45. The equation $y = 3\cos(2x - \pi)$ is of the form $y = A\cos(Bx - C)$ with A = 3, and B = 2, and $C = \pi$. Thus, the amplitude is $|A| = |3| = 3$. The period is $\frac{2\pi}{B} = \frac{2\pi}{2} = \pi$. The phase shift is $\frac{C}{B} = \frac{\pi}{2}$.

The quarter-period is $\frac{\pi}{4}$. The cycle begins at $x = \frac{\pi}{2}$.

Add quarter-periods to generate x-values for the key points.

$$x = \frac{\pi}{2}$$

$$x = \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$$

$$x = \frac{3\pi}{4} + \frac{\pi}{4} = \pi$$

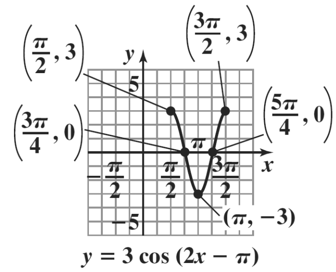
$$x = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$$

$$x = \frac{5\pi}{4} + \frac{\pi}{4} = \frac{3\pi}{2}$$

Evaluate the function at each value of x.

x	coordinates
$\frac{\pi}{2}$	$\left(\frac{\pi}{2}, 3\right)$
$\frac{3\pi}{4}$	$\left(\frac{3\pi}{4}, 0\right)$
π	$(\pi, -3)$
$\frac{5\pi}{4}$	$\left(\frac{5\pi}{4}, 0\right)$
$\frac{3\pi}{2}$	$\left(\frac{3\pi}{2}, 3\right)$

Connect the five points with a smooth curve and graph one complete cycle of the given function



46. The equation $y = 4\cos(2x - \pi)$ is of the form $y = A\cos(Bx - C)$ with A = 4, and B = 2, and $C = \pi$. Thus, the amplitude is $|A| = |4| = 4$. The period is $\frac{2\pi}{B} = \frac{2\pi}{2} = \pi$. The phase shift is $\frac{C}{B} = \frac{\pi}{2}$.

The quarter-period is $\frac{\pi}{4}$. The cycle begins at $x = \frac{\pi}{2}$.

Add quarter-periods to generate x-values for the key points.

$$x = \frac{\pi}{2}$$

$$x = \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$$

$$x = \frac{3\pi}{4} + \frac{\pi}{4} = \pi$$

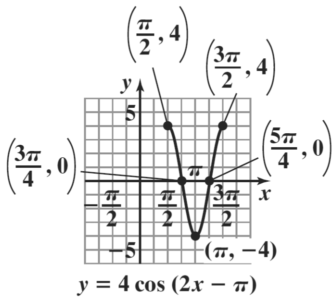
$$x = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$$

$$x = \frac{5\pi}{4} + \frac{\pi}{4} = \frac{3\pi}{2}$$

Evaluate the function at each value of x.

x	coordinates
$\frac{\pi}{2}$	$\left(\frac{\pi}{2}, 4\right)$
$\frac{3\pi}{4}$	$\left(\frac{3\pi}{4}, 0\right)$
π	$(\pi, -4)$
$\frac{5\pi}{4}$	$\left(\frac{5\pi}{4}, 0\right)$
$\frac{3\pi}{2}$	$\left(\frac{3\pi}{2}, 4\right)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



47. $y = \frac{1}{2}\cos\left(3x + \frac{\pi}{2}\right) = \frac{1}{2}\cos\left(3x - \left(-\frac{\pi}{2}\right)\right)$

The equation $y = \frac{1}{2}\cos\left(3x - \left(-\frac{\pi}{2}\right)\right)$ is of the form $y = A\cos(Bx - C)$ with $A = \frac{1}{2}$, and $B = 3$, and $C = -\frac{\pi}{2}$. Thus, the amplitude is $|A| = \left|\frac{1}{2}\right| = \frac{1}{2}$. The period is $\frac{2\pi}{B} = \frac{2\pi}{3}$. The phase shift is $\frac{C}{B} = \frac{-\frac{\pi}{2}}{3} = -\frac{\pi}{2}\cdot\frac{1}{3} = -\frac{\pi}{6}$. The quarter-period is $\frac{\frac{2\pi}{3}}{4} = \frac{2\pi}{3}\cdot\frac{1}{4} = \frac{\pi}{6}$. The cycle begins at $x = -\frac{\pi}{6}$. Add quarter-periods to generate x-values for the key points.

$x = -\frac{\pi}{6}$

$x = -\frac{\pi}{6} + \frac{\pi}{6} = 0$

$x = 0 + \frac{\pi}{6} = \frac{\pi}{6}$

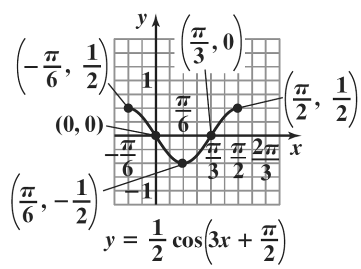
$x = \frac{\pi}{6} + \frac{\pi}{6} = \frac{\pi}{3}$

$x = \frac{\pi}{3} + \frac{\pi}{6} = \frac{\pi}{2}$

Evaluate the function at each value of x.

x	coordinates
$-\frac{\pi}{6}$	$\left(-\frac{\pi}{6}, \frac{1}{2}\right)$
0	(0, 0)
$\frac{\pi}{6}$	$\left(\frac{\pi}{6}, -\frac{1}{2}\right)$
$\frac{\pi}{3}$	$\left(\frac{\pi}{3}, 0\right)$
$\frac{\pi}{2}$	$\left(\frac{\pi}{2}, \frac{1}{2}\right)$

Connect the five points with a smooth curve and graph one complete cycle of the given function



48. $y = \frac{1}{2}\cos(2x + \pi) = \frac{1}{2}\cos(2x - (-\pi))$

The equation $y = \frac{1}{2}\cos(2x - (-\pi))$ is of the form $y = A\cos(Bx - C)$ with $A = \frac{1}{2}$, and $B = 2$, and $C = -\pi$. Thus, the amplitude is $|A| = \left|\frac{1}{2}\right| = \frac{1}{2}$. The period is $\frac{2\pi}{B} = \frac{2\pi}{2} = \pi$. The phase shift is $\frac{C}{B} = \frac{-\pi}{2} = -\frac{\pi}{2}$. The quarter-period is $\frac{\pi}{4}$. The cycle begins at $x = -\frac{\pi}{2}$. Add quarter-periods to generate x-values for the key points.

$x = -\frac{\pi}{2}$

$x = -\frac{\pi}{2} + \frac{\pi}{4} = -\frac{\pi}{4}$

$x = -\frac{\pi}{4} + \frac{\pi}{4} = 0$

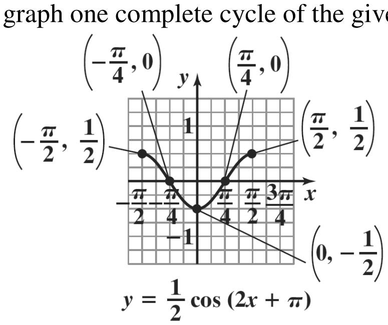
$x = 0 + \frac{\pi}{4} = \frac{\pi}{4}$

$x = \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$

Evaluate the function at each value of x.

x	coordinates
$-\frac{\pi}{2}$	$\left(-\frac{\pi}{2}, \frac{1}{2}\right)$
$-\frac{\pi}{4}$	$\left(-\frac{\pi}{4}, 0\right)$
0	$\left(0, -\frac{1}{2}\right)$
$\frac{\pi}{4}$	$\left(\frac{\pi}{4}, 0\right)$
$\frac{\pi}{2}$	$\left(\frac{\pi}{2}, \frac{1}{2}\right)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



49. The equation $y = -3\cos\left(2x - \frac{\pi}{2}\right)$ is of the form $y = A\cos(Bx - C)$ with $A = -3$, and $B = 2$, and $C = \frac{\pi}{2}$. Thus, the amplitude is $|A| = |-3| = 3$. The period is $\frac{2\pi}{B} = \frac{2\pi}{2} = \pi$. The phase shift is $\frac{C}{B} = \frac{\frac{\pi}{2}}{2} = \frac{\pi}{2} \cdot \frac{1}{2} = \frac{\pi}{4}$.

The quarter-period is $\frac{\pi}{4}$. The cycle begins at $x = \frac{\pi}{4}$. Add quarter-periods to generate x-values for the key points.

$x = \frac{\pi}{4}$

$x = \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$

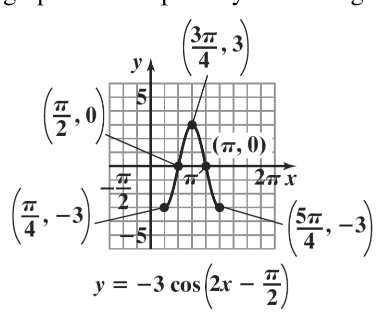
$x = \frac{3\pi}{4} + \frac{\pi}{4} = \pi$

$x = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$

Evaluate the function at each value of x.

x	coordinates
$\frac{\pi}{4}$	$\left(\frac{\pi}{4}, -3\right)$
$\frac{\pi}{2}$	$\left(\frac{\pi}{2}, 0\right)$
$\frac{3\pi}{4}$	$\left(\frac{3\pi}{4}, 3\right)$
π	$(\pi, 0)$
$\frac{5\pi}{4}$	$\left(\frac{5\pi}{4}, -3\right)$

Connect the five points with a smooth curve and graph one complete cycle of the given function



50. The equation $y = -4\cos\left(2x - \frac{\pi}{2}\right)$ is of the form $y = A\cos(Bx - C)$ with $A = -4$, and $B = 2$, and $C = \frac{\pi}{2}$. Thus, the amplitude is $|A| = |-4| = 4$. The period is $\frac{2\pi}{B} = \frac{2\pi}{2} = \pi$. The phase shift is $\frac{C}{B} = \frac{\frac{\pi}{2}}{2} = \frac{\pi}{2}\cdot\frac{1}{2} = \frac{\pi}{4}$. The quarter-period is $\frac{\pi}{4}$. The cycle begins at $x = \frac{\pi}{4}$. Add quarter-periods to generate x-values for the key points.

$x = \frac{\pi}{4}$

$x = \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$

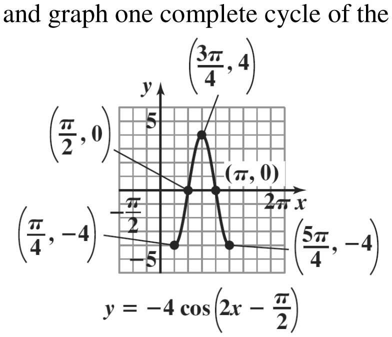
$x = \frac{3\pi}{4} + \frac{\pi}{4} = \pi$

$x = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$

Evaluate the function at each value of x.

x	coordinates
$\frac{\pi}{4}$	$\left(\frac{\pi}{4}, -4\right)$
$\frac{\pi}{2}$	$\left(\frac{\pi}{2}, 0\right)$
$\frac{3\pi}{4}$	$\left(\frac{3\pi}{4}, 4\right)$
π	$(\pi, 0)$
$\frac{5\pi}{4}$	$\left(\frac{5\pi}{4}, -4\right)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



51. $y = 2\cos(2\pi x + 8\pi) = 2\cos(2\pi x - (-8\pi))$

The equation $y = 2\cos(2\pi x - (-8\pi))$ is of the form $y = A\cos(Bx - C)$ with $A = 2$, $B = 2\pi$, and $C = -8\pi$. Thus, the amplitude is $|A| = |2| = 2$. The period is $\frac{2\pi}{B} = \frac{2\pi}{2\pi} = 1$. The phase shift is $\frac{C}{B} = \frac{-8\pi}{2\pi} = -4$. The quarter-period is $\frac{1}{4}$. The cycle begins at $x = -4$. Add quarter-periods to generate x-values for the key points.

$x = -4$

$x = -4 + \frac{1}{4} = -\frac{15}{4}$

$x = -\frac{15}{4} + \frac{1}{4} = -\frac{7}{2}$

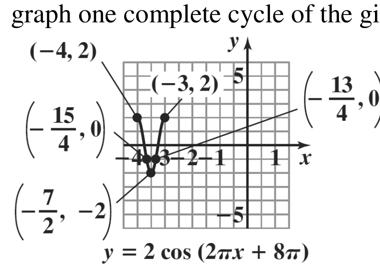
$x = -\frac{7}{2} + \frac{1}{4} = -\frac{13}{4}$

$x = -\frac{13}{4} + \frac{1}{4} = -3$

Evaluate the function at each value of x.

x	coordinates
–4	(–4, 2)
$-\frac{15}{4}$	$\left(-\frac{15}{4}, 0\right)$
$-\frac{7}{2}$	$\left(-\frac{7}{2}, -2\right)$
$-\frac{13}{4}$	$\left(-\frac{13}{4}, 0\right)$
–3	(–3, 2)

Connect the five points with a smooth curve and graph one complete cycle of the given function



52. $y = 3\cos(2\pi x + 4\pi) = 3\cos(2\pi x - (-4\pi))$

The equation $y = 3\cos(2\pi x - (-4\pi))$ is of the form $y = A\cos(Bx - C)$ with $A = 3$, and $B = 2\pi$, and $C = -4\pi$. Thus, the amplitude is $|A| = |3| = 3$. The period is $\frac{2\pi}{B} = \frac{2\pi}{2\pi} = 1$. The phase shift is $\frac{C}{B} = \frac{-4\pi}{2\pi} = -2$. The quarter-period is $\frac{1}{4}$. The cycle begins at $x = -2$. Add quarter-periods to generate x-values for the key points.

$x = -2$

$x = -2 + \frac{1}{4} = -\frac{7}{4}$

$x = -\frac{7}{4} + \frac{1}{4} = -\frac{3}{2}$

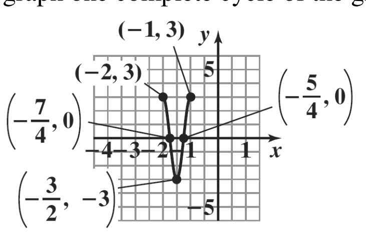
$x = -\frac{3}{2} + \frac{1}{4} = -\frac{5}{4}$

$x = -\frac{5}{4} + \frac{1}{4} = -1$

Evaluate the function at each value of x.

x	coordinates
-2	$(-2, 3)$
$-\frac{7}{4}$	$\left(-\frac{7}{4}, 0\right)$
$-\frac{3}{2}$	$\left(-\frac{3}{2}, -3\right)$
$-\frac{5}{4}$	$\left(-\frac{5\pi}{4}, 0\right)$
-1	$(-1, 3)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



$y = 3\cos(2\pi x + 4\pi)$

53. The graph of $y = \sin x + 2$ is the graph of $y = \sin x$ shifted up 2 units upward. The period for both functions is 2π. The quarter-period is $\frac{2\pi}{4}$ or $\frac{\pi}{2}$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{\pi}{2} = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{2} = \pi$

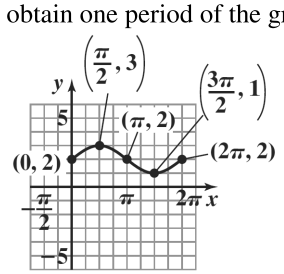
$x = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$

$x = \frac{3\pi}{2} + \frac{\pi}{2} = 2\pi$

Evaluate the function at each value of x.

x	$y = \sin x + 2$	coordinates
0	$y = \sin 0 + 2$ $= 0 + 2 = 2$	$(0, 2)$
$\frac{\pi}{2}$	$y = \sin\frac{\pi}{2} + 2$ $= 1 + 2 = 3$	$\left(\frac{\pi}{2}, 3\right)$
π	$y = \sin\pi + 2$ $= 0 + 2 = 2$	$(\pi, 2)$
$\frac{3\pi}{2}$	$y = \sin\frac{3\pi}{2} + 2$ $= -1 + 2 = 1$	$\left(\frac{3\pi}{2}, 1\right)$
2π	$y = \sin 2\pi + 2$ $= 0 + 2 = 2$	$(2\pi, 2)$

By connecting the points with a smooth curve we obtain one period of the graph.



$y = \sin x + 2$

54. The graph of $y = \sin x - 2$ is the graph of $y = \sin x$ shifted 2 units downward. The period for both functions is 2π. The quarter-period is $\frac{2\pi}{4}$ or $\frac{\pi}{2}$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{\pi}{2} = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{2} = \pi$

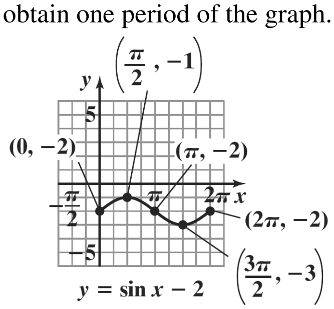
$x = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$

$x = \frac{3\pi}{2} + \frac{\pi}{2} = 2\pi$

Evaluate the function at each value of x.

x	$y = \sin x - 2$	coordinates
0	$y = \sin 0 - 2 = 0 - 2 = -2$	(0, –2)
$\frac{\pi}{2}$	$y = \sin\frac{\pi}{2} - 2 = 1 - 2 = -1$	$\left(\frac{\pi}{4}, -1\right)$
π	$y = \sin\pi - 2 = 0 - 2 = -2$	$(\pi, -2)$
$\frac{3\pi}{2}$	$y = \sin\frac{3\pi}{2} - 2 = -1 - 2 = -3$	$\left(\frac{3\pi}{2}, -3\right)$
2π	$y = \sin 2\pi - 2 = 0 - 2 = -2$	$(2\pi, -2)$

By connecting the points with a smooth curve we obtain one period of the graph.



55. The graph of $y = \cos x - 3$ is the graph of $y = \cos x$ shifted 3 units downward. The period for both functions is 2π. The quarter-period is $\frac{2\pi}{4}$ or $\frac{\pi}{2}$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{\pi}{2} = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{2} = \pi$

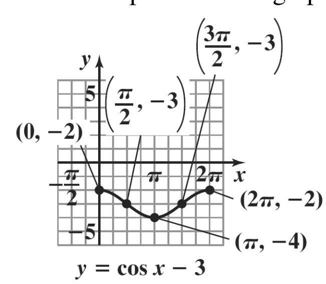
$x = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$

$x = \frac{3\pi}{2} + \frac{\pi}{2} = 2\pi$

Evaluate the function at each value of x.

x	$y = \cos x - 3$	coordinates
0	$y = \cos 0 - 3$ $= 1 - 3 = -2$	(0, –2)
$\frac{\pi}{2}$	$y = \cos\frac{\pi}{2} - 3$ $= 0 - 3 = -3$	$\left(\frac{\pi}{2}, -3\right)$
π	$y = \cos\pi - 3$ $= -1 - 3 = -4$	$(\pi, -4)$
$\frac{3\pi}{2}$	$y = \cos\frac{3\pi}{2} - 3$ $= 0 - 3 = -3$	$\left(\frac{3\pi}{2}, -3\right)$
2π	$y = \cos 2\pi - 3$ $= 1 - 3 = -2$	$(2\pi, -2)$

By connecting the points with a smooth curve we obtain one period of the graph.



56. The graph of $y = \cos x + 3$ is the graph of $y = \cos x$ shifted 3 units upward. The period for both functions is 2π. The quarter-period is $\frac{2\pi}{4}$ or $\frac{\pi}{2}$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{\pi}{2} = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{2} = \pi$

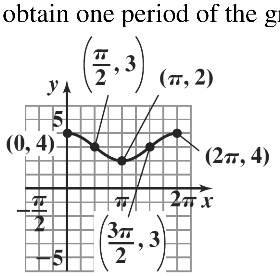
$x = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$

$x = \frac{3\pi}{2} + \frac{\pi}{2} = 2\pi$

Evaluate the function at each value of x.

x	$y = \cos x + 3$	coordinates
0	$y = \cos 0 + 3 = 1 + 3 = 4$	(0, 4)
$\frac{\pi}{2}$	$y = \cos\frac{\pi}{2} + 3 = 0 + 3 = 3$	$\left(\frac{\pi}{2}, 3\right)$
π	$y = \cos\pi + 3 = -1 + 3 = 2$	$(\pi, 2)$
$\frac{3\pi}{2}$	$y = \cos\frac{3\pi}{2} + 3 = 0 + 3 = 3$	$\left(\frac{3\pi}{2}, 3\right)$
2π	$y = \cos 2\pi + 3 = 1 + 3 = 4$	$(2\pi, 4)$

By connecting the points with a smooth curve we obtain one period of the graph.



$y = \cos x + 3$

57. The graph of $y = 2\sin\frac{1}{2}x + 1$ is the graph of $y = 2\sin\frac{1}{2}x$ shifted one unit upward. The amplitude for both functions is $|\,2\,| = 2$. The period for both functions is $\frac{2\pi}{\frac{1}{2}} = 2\pi \cdot 2 = 4\pi$. The quarter-period is $\frac{4\pi}{4} = \pi$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \pi = \pi$

$x = \pi + \pi = 2\pi$

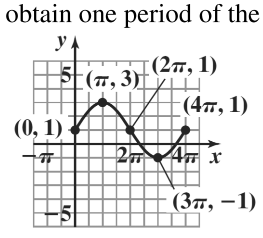
$x = 2\pi + \pi = 3\pi$

$x = 3\pi + \pi = 4\pi$

Evaluate the function at each value of x.

x	$y = 2\sin\frac{1}{2}x + 1$	coordinates
0	$y = 2\sin\left(\frac{1}{2}\cdot 0\right) + 1$ $= 2\sin 0 + 1$ $= 2\cdot 0 + 1 = 0 + 1 = 1$	(0, 1)
π	$y = 2\sin\left(\frac{1}{2}\cdot \pi\right) + 1$ $= 2\sin\frac{\pi}{2} + 1$ $= 2\cdot 1 + 1 = 2 + 1 = 3$	$(\pi, 3)$
2π	$y = 2\sin\left(\frac{1}{2}\cdot 2\pi\right) + 1$ $= 2\sin\pi + 1$ $= 2\cdot 0 + 1 = 0 + 1 = 1$	$(2\pi, 1)$
3π	$y = 2\sin\left(\frac{1}{2}\cdot 3\pi\right) + 1$ $= 2\sin\frac{3\pi}{2} + 1$ $= 2\cdot(-1) + 1$ $= -2 + 1 = -1$	$(3\pi, -1)$
4π	$y = 2\sin\left(\frac{1}{2}\cdot 4\pi\right) + 1$ $= 2\sin 2\pi + 1$ $= 2\cdot 0 + 1 = 0 + 1 = 1$	$(4\pi, 1)$

By connecting the points with a smooth curve we obtain one period of the graph.



$y = 2\sin\frac{1}{2}x + 1$

58. The graph of $y = 2\cos\frac{1}{2}x + 1$ is the graph of $y = 2\cos\frac{1}{2}x$ shifted one unit upward. The amplitude for both functions is $|2| = 2$. The period for both functions is $\frac{2\pi}{\frac{1}{2}} = 2\pi \cdot 2 = 4\pi$. The quarter-period is $\frac{4\pi}{4} = \pi$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \pi = \pi$

$x = \pi + \pi = 2\pi$

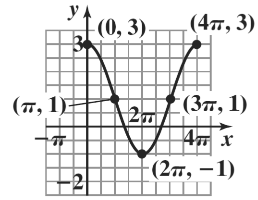
$x = 2\pi + \pi = 3\pi$

$x = 3\pi + \pi = 4\pi$

Evaluate the function at each value of x.

x	$y = 2\cos\frac{1}{2}x + 1$	coordinates
0	$y = 2\cos\left(\frac{1}{2}\cdot 0\right) + 1$ $= 2\cos 0 + 1$ $= 2\cdot 1 + 1 = 2 + 1 = 3$	(0, 3)
π	$y = 2\cos\left(\frac{1}{2}\cdot \pi\right) + 1$ $= 2\cos\frac{\pi}{2} + 1$ $= 2\cdot 0 + 1 = 0 + 1 = 1$	$(\pi, 1)$
2π	$y = 2\cos\left(\frac{1}{2}\cdot 2\pi\right) + 1$ $= 2\cos\pi + 1$ $= 2\cdot(-1) + 1 = -2 + 1 = -1$	$(2\pi, -1)$
3π	$y = 2\cos\left(\frac{1}{2}\cdot 3\pi\right) + 1$ $= 2\cdot 0 + 1 = 0 + 1 = 1$	$(3\pi, 1)$
4π	$y = 2\cos\left(\frac{1}{2}\cdot 4\pi\right) + 1$ $= 2\cos 2\pi + 1$ $= 2\cdot 1 + 1 = 2 + 1 = 3$	$(4\pi, 3)$

By connecting the points with a smooth curve we obtain one period of the graph.



$y = 2\cos\frac{1}{2}x + 1$

59. The graph of $y = -3\cos 2\pi x + 2$ is the graph of $y = -3\cos 2\pi x$ shifted 2 units upward. The amplitude for both functions is $|-3| = 3$. The period for both functions is $\frac{2\pi}{2\pi} = 1$. The quarter-period is $\frac{1}{4}$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{1}{4} = \frac{1}{4}$

$x = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

$x = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$

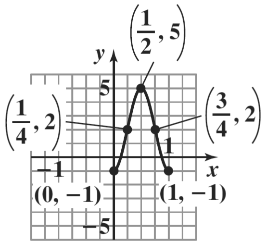
$x = \frac{3}{4} + \frac{1}{4} = 1$

Evaluate the function at each value of x.

x	$y = -3\cos 2\pi x + 2$	coordinates
0	$y = -3\cos(2\pi\cdot 0) + 2$ $= -3\cos 0 + 2$ $= -3\cdot 1 + 2$ $= -3 + 2 = -1$	(0, –1)
$\frac{1}{4}$	$y = -3\cos\left(2\pi\cdot\frac{1}{4}\right) + 2$ $= -3\cos\frac{\pi}{2} + 2$ $= -3\cdot 0 + 2$ $= 0 + 2 = 2$	$\left(\frac{1}{4}, 2\right)$
$\frac{1}{2}$	$y = -3\cos\left(2\pi\cdot\frac{1}{2}\right) + 2$ $= -3\cos\pi + 2$ $= -3\cdot(-1) + 2$ $= 3 + 2 = 5$	$\left(\frac{1}{2}, 5\right)$

$\frac{3}{4}$	$y=-3\cos\left(2\pi\cdot\frac{3}{4}\right)+2$ $=-3\cos\frac{3\pi}{2}+2$ $=-3\cdot 0+2$ $=0+2=2$	$\left(\frac{3}{4}, 2\right)$
1	$y=-3\cos(2\pi\cdot 1)+2$ $=-3\cos 2\pi+2$ $=-3\cdot 1+2$ $=-3+2=-1$	$(1, -1)$

By connecting the points with a smooth curve we obtain one period of the graph.



$y = -3\cos 2\pi x + 2$

60. The graph of $y=-3\sin 2\pi x+2$ is the graph of $y=-3\sin 2\pi x$ shifted two units upward. The amplitude for both functions is $|A|=|-3|=3$. The period for both functions is $\frac{2\pi}{2\pi}=1$. The quarter-period is $\frac{1}{4}$. The cycle begins at $x = 0$. Add quarter–periods to generate x-values for the key points.

$x=0$

$x=0+\frac{1}{4}=\frac{1}{4}$

$x=\frac{1}{4}+\frac{1}{4}=\frac{1}{2}$

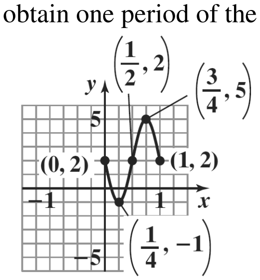
$x=\frac{1}{2}+\frac{1}{4}=\frac{3}{4}$

$x=\frac{3}{4}+\frac{1}{4}=1$

Evaluate the function at each value of x.

x	$y=-3\sin 2\pi x+2$	coordinates
0	$y=-3\sin(2\pi\cdot 0)+2$ $=-3\sin 0+2$ $=-3\cdot 0+2=0+2=2$	(0, 2)
$\frac{1}{4}$	$y=-3\sin\left(2\pi\cdot\frac{1}{4}\right)+2$ $=-3\sin\frac{\pi}{2}+2$ $=-3\cdot 1+2=-3+2=-1$	$\left(\frac{1}{4}, -1\right)$
$\frac{1}{2}$	$y=-3\sin\left(2\pi\cdot\frac{1}{2}\right)+2$ $=-3\sin\pi+2$ $=-3\cdot 0+2=0+2=2$	$\left(\frac{1}{2}, 2\right)$
$\frac{3}{4}$	$y=-3\sin\left(2\pi\cdot\frac{3}{4}\right)+2$ $=-3\sin\frac{3\pi}{2}+2$ $=-3\cdot(-1)+2=3+2=5$	$\left(\frac{3}{4}, 5\right)$
1	$y=-3\sin(2\pi\cdot 1)+2$ $=-3\sin 2\pi+2$ $=-3\cdot 0+2=0+2=2$	(1, 2)

By connecting the points with a smooth curve we obtain one period of the graph.



$y = -3\sin 2\pi x + 2$

61. Using $y=A\cos Bx$ the amplitude is 3 and $A=3$, The period is 4π and thus

$B=\frac{2\pi}{\text{period}}=\frac{2\pi}{4\pi}=\frac{1}{2}$

$y=A\cos Bx$

$y=3\cos\frac{1}{2}x$

62. Using $y = A\sin Bx$ the amplitude is 3 and $A = 3$,
The period is 4π and thus

$$B = \frac{2\pi}{\text{period}} = \frac{2\pi}{4\pi} = \frac{1}{2}$$
$$y = A\sin Bx$$
$$y = 3\sin\frac{1}{2}x$$

63. Using $y = A\sin Bx$ the amplitude is 2 and $A = -2$,
The period is π and thus

$$B = \frac{2\pi}{\text{period}} = \frac{2\pi}{\pi} = 2$$
$$y = A\sin Bx$$
$$y = -2\sin 2x$$

64. Using $y = A\cos Bx$ the amplitude is 2 and $A = -2$,
The period is 4π and thus

$$B = \frac{2\pi}{\text{period}} = \frac{2\pi}{\pi} = 2$$
$$y = A\cos Bx$$
$$y = -2\cos 2x$$

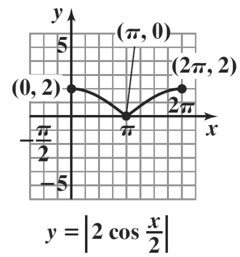
65. Using $y = A\sin Bx$ the amplitude is 2 and $A = 2$,
The period is 4 and thus

$$B = \frac{2\pi}{\text{period}} = \frac{2\pi}{4} = \frac{\pi}{2}$$
$$y = A\sin Bx$$
$$y = 2\sin\left(\frac{\pi}{2}x\right)$$

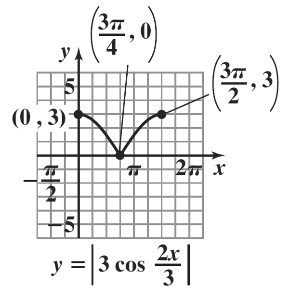
66. Using $y = A\cos Bx$ the amplitude is 2 and $A = 2$,
The period is 4 and thus

$$B = \frac{2\pi}{\text{period}} = \frac{2\pi}{4} = \frac{\pi}{2}$$
$$y = A\cos Bx$$
$$y = 2\cos\left(\frac{\pi}{2}x\right)$$

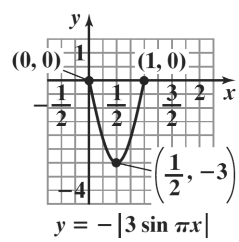
67.



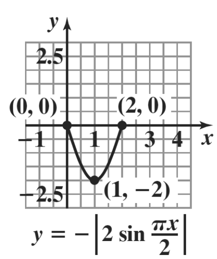
68.



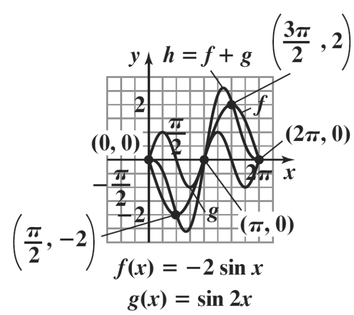
69.



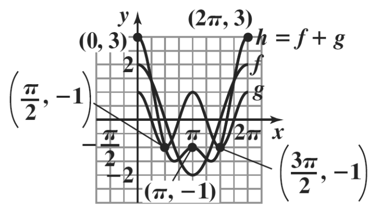
70.



71.

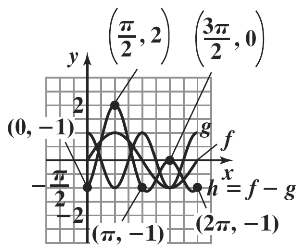


72.



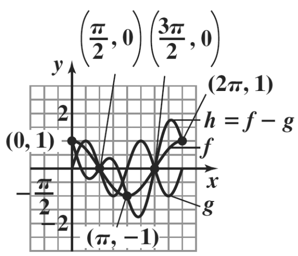
$f(x) = 2\cos x$
$g(x) = \cos 2x$

73.



$f(x) = \sin x$
$g(x) = \cos 2x$

74.



$f(x) = \cos x$
$g(x) = \sin 2x$

75. The period of the physical cycle is 33 days.

76. The period of the emotional cycle is 28 days.

77. The period of the intellectual cycle is 23 days.

78. In the month of February, the physical cycle is at a minimum on February 18. Thus, the author should not run in a marathon on February 18.

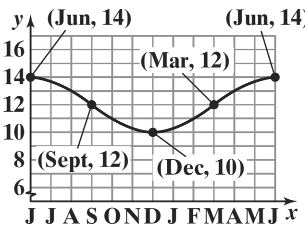
79. In the month of March, March 21 would be the best day to meet an on-line friend for the first time, because the emotional cycle is at a maximum.

80. In the month of February, the intellectual cycle is at a maximum on February 11. Thus, the author should begin writing the on February 11.

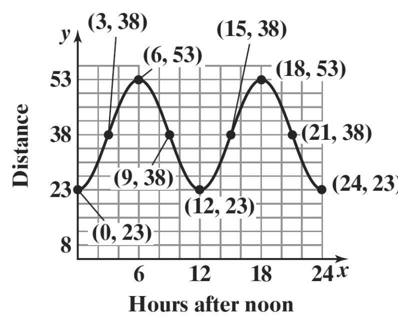
81. Answers may vary.

82. Answers may vary.

83. The information gives the five key point of the graph.
(0, 14) corresponds to June,
(3, 12) corresponds to September,
(6, 10) corresponds to December,
(9, 12) corresponds to March,
(12, 14) corresponds to June
By connecting the five key points with a smooth curve we graph the information from June of one year to June of the following year.



84. The information gives the five key points of the graph.
(0, 23) corresponds to Noon,
(3, 38) corresponds to 3 P.M.,
(6, 53) corresponds to 6 P.M.,
(9, 38) corresponds to 9 P.M.,
(12, 23) corresponds to Midnight.
By connecting the five key points with a smooth curve we graph information from noon to midnight. Extend the graph one cycle to the right to graph the information for $0 \le x \le 24$.



85. The function $y = 3\sin\frac{2\pi}{365}(x-79)+12$ is of the form $y = A\sin B\left(x-\frac{C}{B}\right)+D$ with $A = 3$ and $B = \frac{2\pi}{365}$.

a. The amplitude is $|A| = |3| = 3$.

b. The period is $\frac{2\pi}{B} = \frac{2\pi}{\frac{2\pi}{365}} = 2\pi\cdot\frac{365}{2\pi} = 365$.

c. The longest day of the year will have the most hours of daylight. This occurs when the sine function equals 1.

$y = 3\sin\frac{2\pi}{365}(x-79)+12$

$y = 3(1)+12$

$y = 15$

There will be 15 hours of daylight.

d. The shortest day of the year will have the least hours of daylight. This occurs when the sine function equals –1.

$y = 3\sin\frac{2\pi}{365}(x-79)+12$

$y = 3(-1)+12$

$y = 9$

There will be 9 hours of daylight.

e. The amplitude is 3. The period is 365. The phase shift is $\frac{C}{B} = 79$. The quarter-period is $\frac{365}{4} = 91.25$. The cycle begins at $x = 79$. Add quarter-periods to find the x-values of the key points.

$x = 79$

$x = 79 + 91.25 = 170.25$

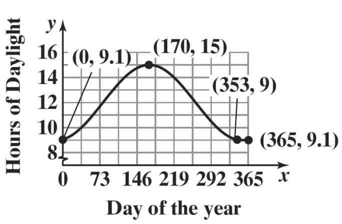
$x = 170.25 + 91.25 = 261.5$

$x = 261.5 + 91.25 = 352.75$

$x = 352.75 + 91.25 = 444$

Because we are graphing for $0 \le x \le 365$, we will evaluate the function for the first four x-values along with $x = 0$ and $x = 365$. Using a calculator we have the following points.
(0, 9.1) (79, 12) (170.25, 15)
(261.5, 12) (352.75, 9) (365, 9.1)

By connecting the points with a smooth curve we obtain one period of the graph, starting on January 1.



86. The function $y = 16\sin\left(\frac{\pi}{6}x - \frac{2\pi}{3}\right)+40$ is in the form $y = A\sin(Bx - C) + D$ with $A = 16$, $B = \frac{\pi}{6}$, and $C = \frac{2\pi}{3}$. The amplitude is $|A| = |16| = 16$. The period is $\frac{2\pi}{B} = \frac{2\pi}{\frac{\pi}{6}} = 2\pi\cdot\frac{6}{\pi} = 12$. The phase shift is $\frac{C}{B} = \frac{\frac{2\pi}{3}}{\frac{\pi}{6}} = \frac{2\pi}{3}\cdot\frac{6}{\pi} = 4$. The quarter-period is $\frac{12}{4} = 3$. The cycle begins at $x = 4$. Add quarter-periods to find the x-values for the key points.

$x = 4$

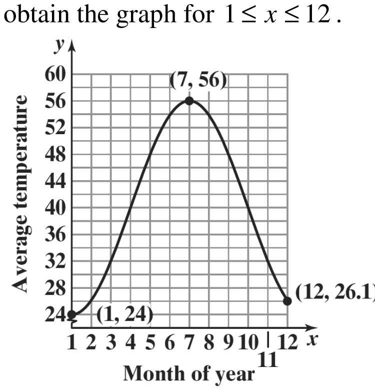
$x = 4 + 3 = 7$

$x = 7 + 3 = 10$

$x = 10 + 3 = 13$

$x = 13 + 3 = 16$

Because we are graphing for $1 \le x \le 12$, we will evaluate the function for the three x-values between 1 and 12, along with $x = 1$ and $x = 12$. Using a calculator we have the following points.
(1, 24) (4, 40) (7, 56) (10, 40) (12, 26.1)
By connecting the points with a smooth curve we obtain the graph for $1 \le x \le 12$.



The highest average monthly temperature is 56° in July.

87. Because the depth of the water ranges from a minimum of 6 feet to a maximum of 12 feet, the curve oscillates about the middle value, 9 feet. Thus, $D = 9$. The maximum depth of the water is 3 feet above 9 feet. Thus, $A = 3$. The graph shows that one complete cycle occurs in 12-0, or 12 hours. The period is 12. Thus,

$$12 = \frac{2\pi}{B}$$
$$12B = 2\pi$$
$$B = \frac{2\pi}{12} = \frac{\pi}{6}$$

Substitute these values into $y = A\cos Bx + D$. The depth of the water is modeled by $y = 3\cos\frac{\pi x}{6} + 9$.

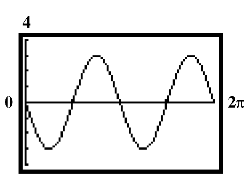
88. Because the depth of the water ranges from a minimum of 3 feet to a maximum of 5 feet, the curve oscillates about the middle value, 4 feet. Thus, $D = 4$. The maximum depth of the water is 1 foot above 4 feet. Thus, $A = 1$. The graph shows that one complete cycle occurs in 12–0, or 12 hours. The period is 12. Thus,

$$12 = \frac{2\pi}{B}$$
$$12B = 2\pi$$
$$B = \frac{2\pi}{12} = \frac{\pi}{6}$$

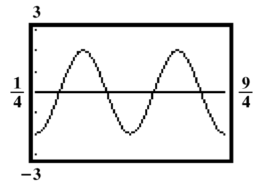
Substitute these values into $y = A\cos Bx + D$. The depth of the water is modeled by $y = \cos\frac{\pi x}{6} + 4$.

89. – 100. Answers may vary.

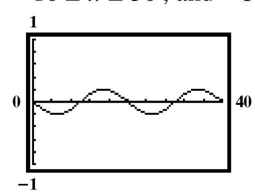
101. The function $y = 3\sin(2x + \pi) = 3\sin(2x - (-\pi))$ is of the form $y = A\sin(Bx - C)$ with $A = 3$, $B = 2$, and $C = -\pi$. The amplitude is $|A| = |3| = 3$. The period is $\frac{2\pi}{B} = \frac{2\pi}{2} = \pi$. The cycle begins at $x = \frac{C}{B} = \frac{-\pi}{2} = -\frac{\pi}{2}$. We choose $-\frac{\pi}{2} \le x \le \frac{3\pi}{2}$, and $-4 \le y \le 4$ for our graph.



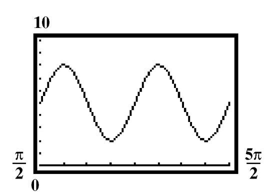
102. The function $y = -2\cos\left(2\pi x - \frac{\pi}{2}\right)$ is of the form $y = A\cos(Bx - C)$ with $A = -2$, $B = 2\pi$, and $C = \frac{\pi}{2}$. The amplitude is $|A| = |-2| = 2$. The period is $\frac{2\pi}{B} = \frac{2\pi}{2\pi} = 1$. The cycle begins at $x = \frac{C}{B} = \frac{\frac{\pi}{2}}{2\pi} = \frac{\pi}{2}\cdot\frac{1}{2\pi} = \frac{1}{4}$. We choose $\frac{1}{4} \le x \le \frac{9}{4}$, and $-3 \le y \le 3$ for our graph.



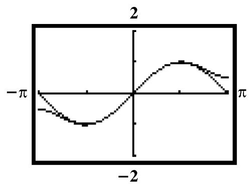
103. The function $y = 0.2\sin\left(\frac{\pi}{10}x + \pi\right) = 0.2\sin\left(\frac{\pi}{10}x - (-\pi)\right)$ is of the form $y = A\sin(Bx - C)$ with $A = 0.2$, $B = \frac{\pi}{10}$, and $C = -\pi$. The amplitude is $|A| = |0.2| = 0.2$. The period is $\frac{2\pi}{B} = \frac{2\pi}{\frac{\pi}{10}} = 2\pi\cdot\frac{10}{\pi} = 20$. The cycle begins at $x = \frac{C}{B} = \frac{-\pi}{\frac{\pi}{10}} = -\pi\cdot\frac{10}{\pi} = -10$. We choose $-10 \le x \le 30$, and $-1 \le y \le 1$ for our graph.



104. The function $y = 3\sin(2x - \pi) + 5$ is of the form $y = A\cos(Bx - C) + D$ with $A = 3$, $B = 2$, $C = \pi$, and $D = 5$. The amplitude is $|A| = |3| = 3$. The period is $\frac{2\pi}{B} = \frac{2\pi}{2} = \pi$. The cycle begins at $x = \frac{C}{B} = \frac{\pi}{2}$. Because $D = 5$, the graph has a vertical shift 5 units upward. We choose $\frac{\pi}{2} \le x \le \frac{5\pi}{2}$, and $0 \le y \le 10$ for our graph.

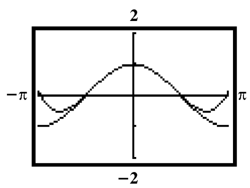


105.



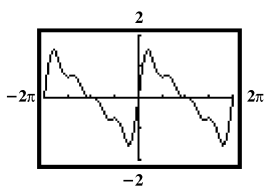
The graphs appear to be the same from $-\dfrac{\pi}{2}$ to $\dfrac{\pi}{2}$.

106.



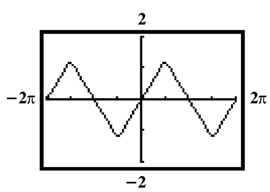
The graphs appear to be the same from $-\dfrac{\pi}{2}$ to $\dfrac{\pi}{2}$.

107.



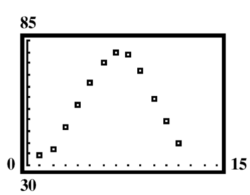
The graph is similar to $y = \sin x$, except the amplitude is greater and the curve is less smooth.

108.

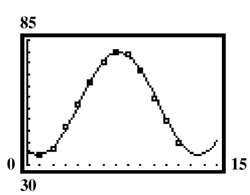


The graph is very similar to $y = \sin x$, except not smooth.

109. a.



b. $y = 22.61\sin(0.50x - 2.04) + 57.17$



110. Answers may vary.

111. makes sense

112. does not make sense; Explanations will vary. Sample explanation: It may be easier to start at the highest point.

113. makes sense

114. makes sense

115. a. Since $A = 3$ and $D = -2$, the maximum will occur at $3 - 2 = 1$ and the minimum will occur at $-3 - 2 = -5$. Thus the range is $[-5, 1]$

Viewing rectangle: $\left[-\dfrac{\pi}{6}, \dfrac{23\pi}{6}, \dfrac{\pi}{6}\right]$ by $[-5, 1, 1]$

b. Since $A = 1$ and $D = -2$, the maximum will occur at $1 - 2 = -1$ and the minimum will occur at $-1 - 2 = -3$. Thus the range is $[-3, -1]$

Viewing rectangle: $\left[-\dfrac{\pi}{6}, \dfrac{7\pi}{6}, \dfrac{\pi}{6}\right]$ by $[-3, -1, 1]$

116. $A = \pi$

$$B = \frac{2\pi}{\text{period}} = \frac{2\pi}{1} = 2\pi$$

$$\frac{C}{B} = \frac{C}{2\pi} = -2$$

$C = -4\pi$

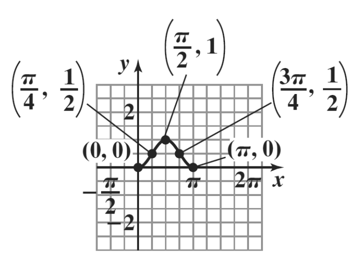
$y = A\cos(Bx - C)$

$y = \pi\cos(2\pi x + 4\pi)$

or

$y = \pi\cos[2\pi(x + 2)]$

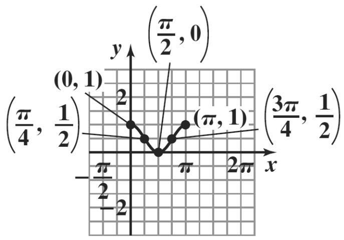
117. $y = \sin^2 x = \dfrac{1}{2} - \dfrac{1}{2}\cos 2x$



$y = \sin^2 x$

or $y = \dfrac{1}{2} - \dfrac{1}{2}\cos 2x$

118. $y = \cos^2 x = \frac{1}{2} + \frac{1}{2}\cos 2x$

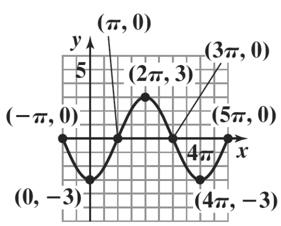


119. Answers may vary.

120.
$$-\frac{\pi}{2} < x + \frac{\pi}{4} < \frac{\pi}{2}$$
$$-\frac{\pi}{2} - \frac{\pi}{4} < x + \frac{\pi}{4} - \frac{\pi}{4} < \frac{\pi}{2} - \frac{\pi}{4}$$
$$-\frac{2\pi}{4} - \frac{\pi}{4} < x < \frac{2\pi}{4} - \frac{\pi}{4}$$
$$-\frac{3\pi}{4} < x < \frac{\pi}{4}$$
$$\left\{x \,\middle|\, -\frac{3\pi}{4} < x < \frac{\pi}{4}\right\} \text{ or } \left(-\frac{3\pi}{4}, \frac{\pi}{4}\right)$$

121. $\dfrac{-\frac{3\pi}{4} + \frac{\pi}{4}}{2} = \dfrac{-\frac{2\pi}{4}}{2} = \dfrac{-\frac{\pi}{2}}{2} = -\dfrac{\pi}{4}$

122. a.



b. The reciprocal function is undefined.